Introduction to Machine Learning

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Slides link:

https://yaoxiangding.github.io/introML-2023/lec7-RL.pdf

Summer 2023

Week 16

Announcements

- **考查方法 Evaluation Method**

  平时成绩：20分 4次 小测 （5分/次）

  报告：70分

  墙报展示：10分

1.报告内容(70分)

Report Types

论文综述 Paper review

算法实践Case study

二选一（select one from the above two）

Slides link:

Reinforcement Learning

- Basics
  - Markov decision process
  - RL with known model
  - RL with unknown model
  - Policy gradient & actor-critic methods
- Deep reinforcement learning
- Integrating learning and planning
- RL from human preference
- Take-home messages

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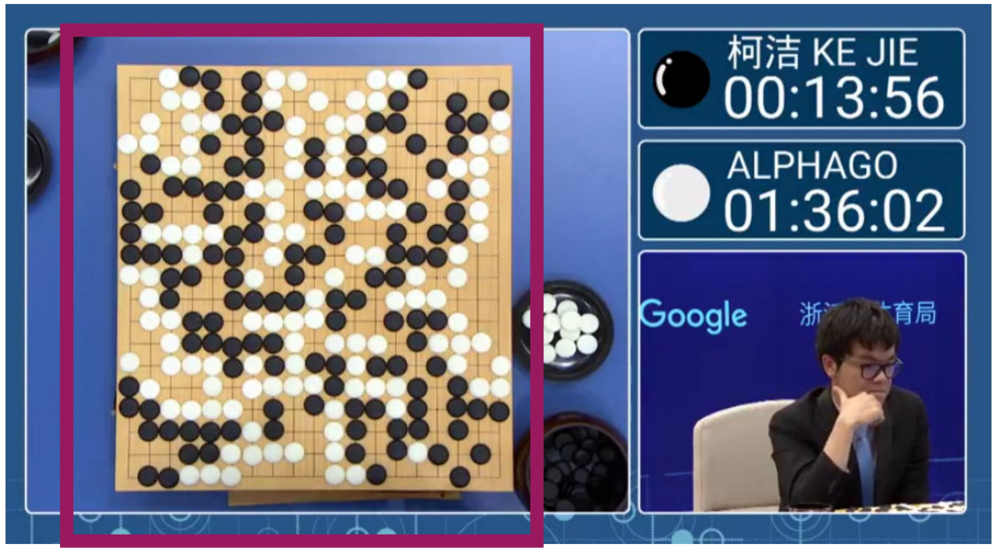
Slides link:

Decision Making

Decision Making



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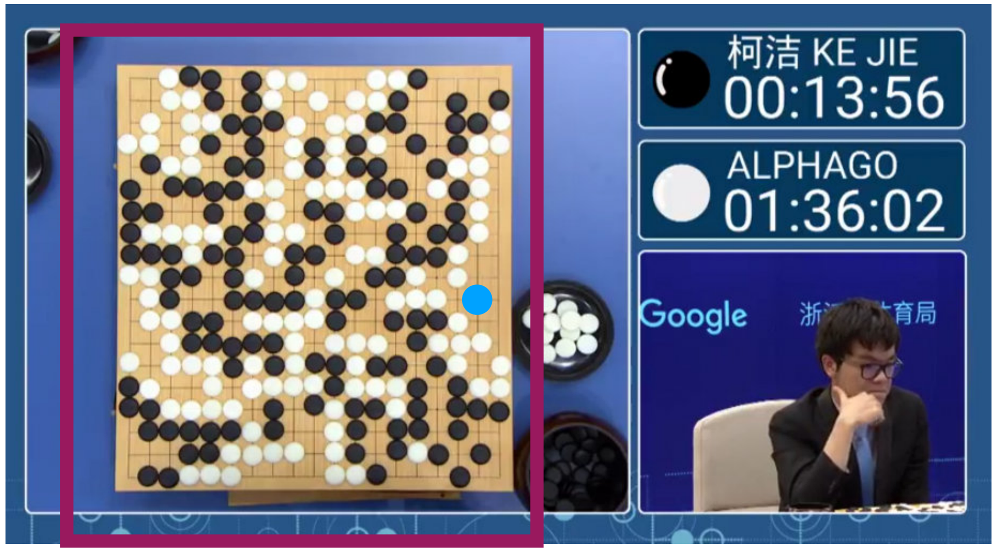
Decision Making



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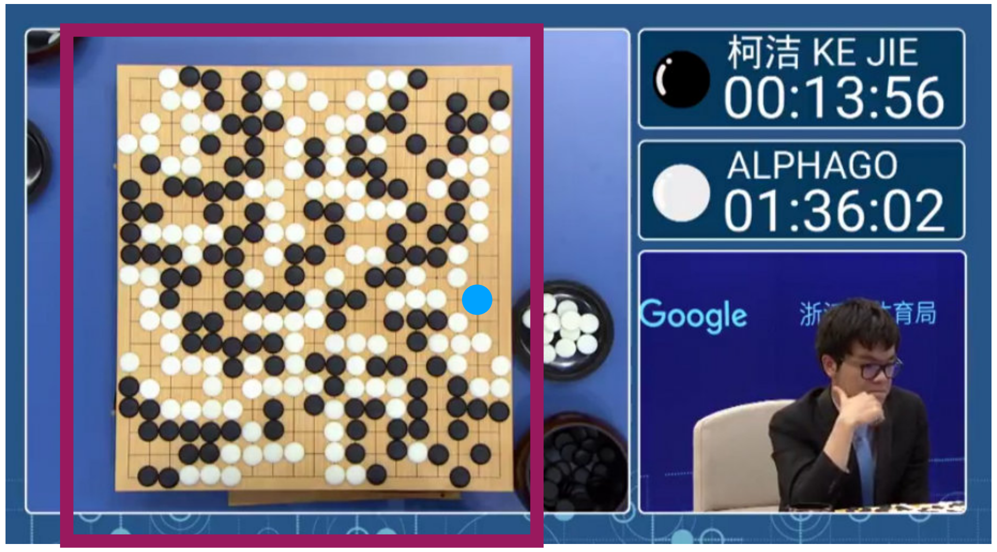

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- Need to choose the corresponding "actions".
- Each action has a utility/cost.

In this lecture,
we given utility a name: reward.

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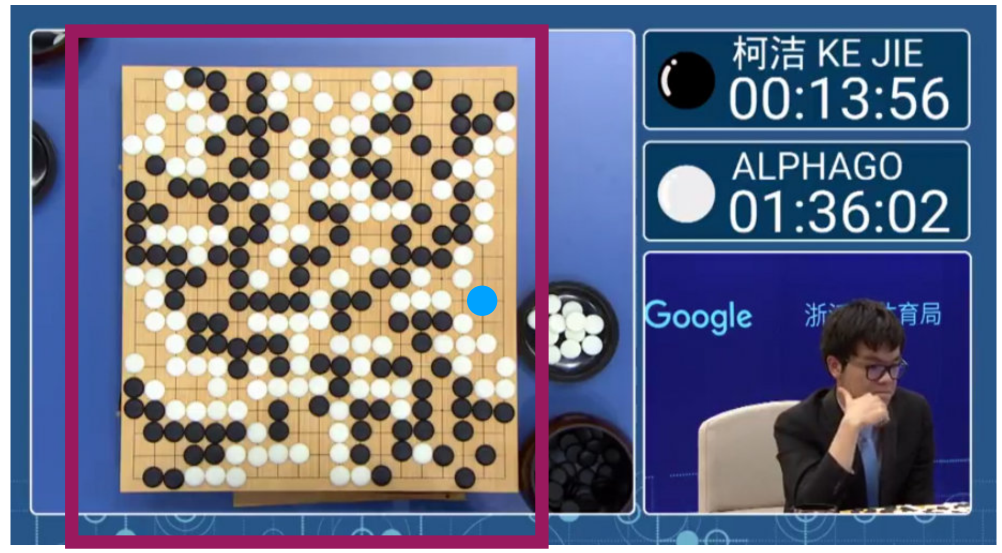

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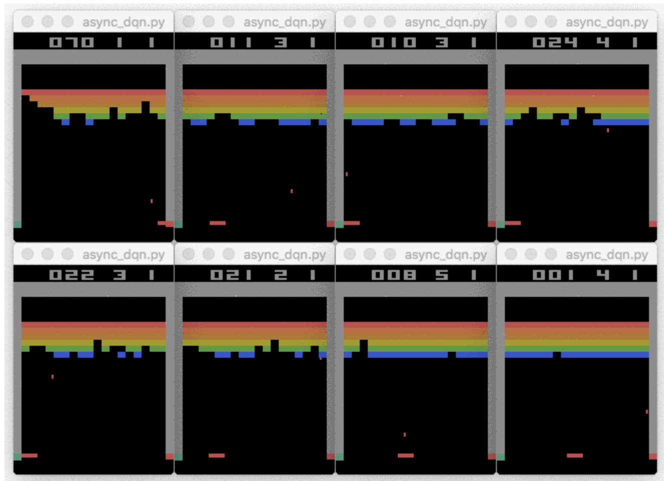
- Target: maximize the total reward in a decision sequence by always choosing the right action.

Decision Making



- Conduct action in any state of an environment.

In most problems, the agent needs to do a sequence of actions w.r.t. a sequence of states.

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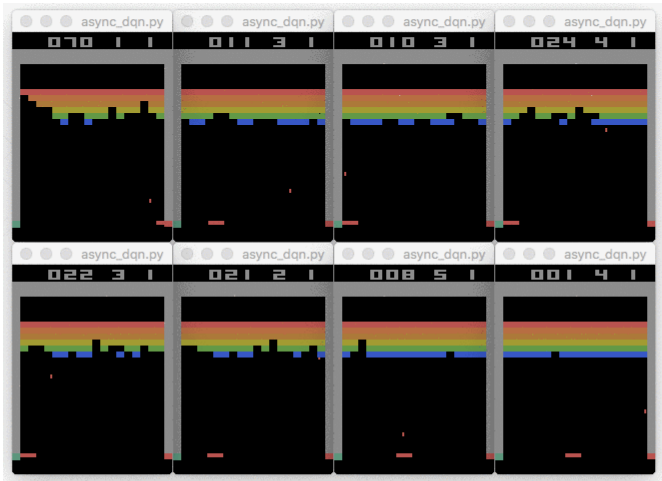

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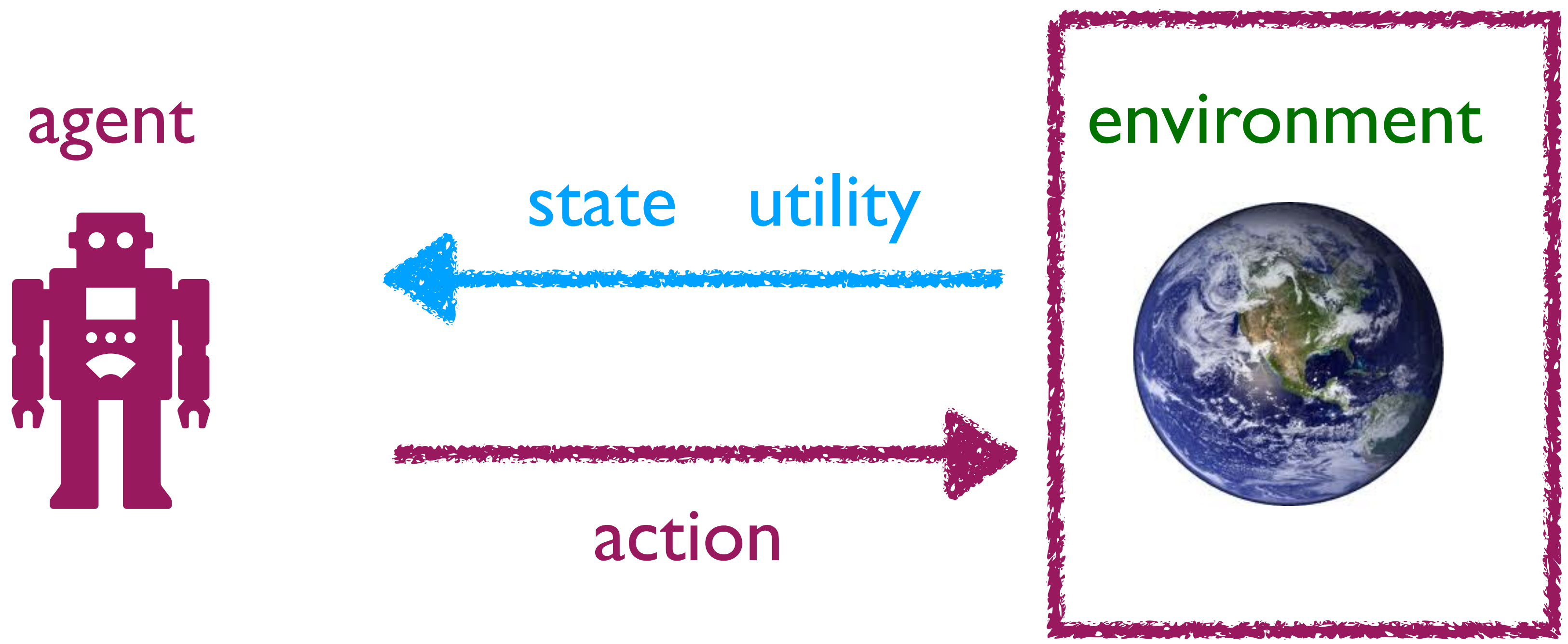
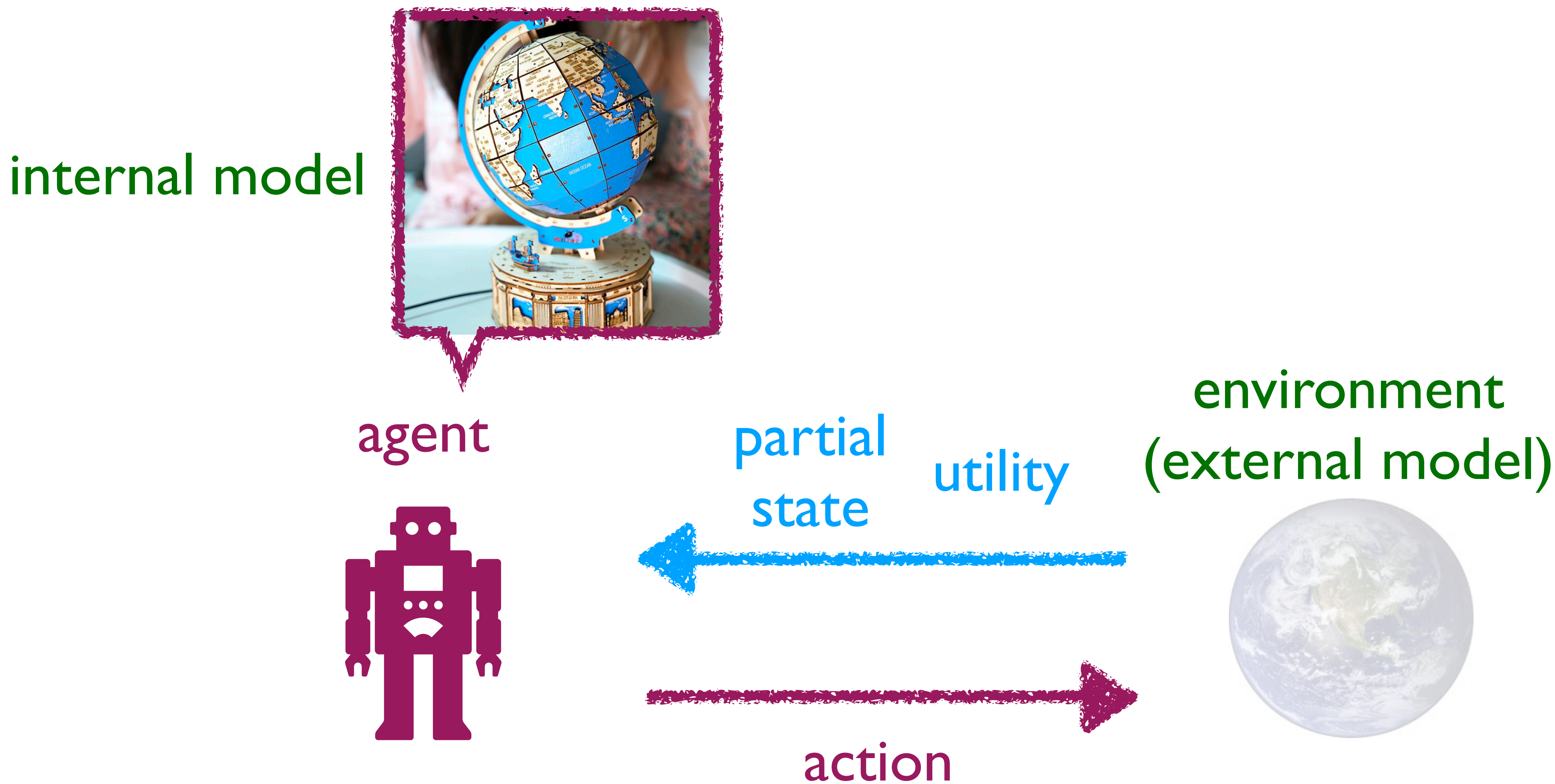
Model of the Environment



To make decisions in the environment, the agent usually needs a model of the environment to know how the things go on.
Where does this model come from?
Given by the problem (external) or built by the agent? (internal)

Internal vs. External Model

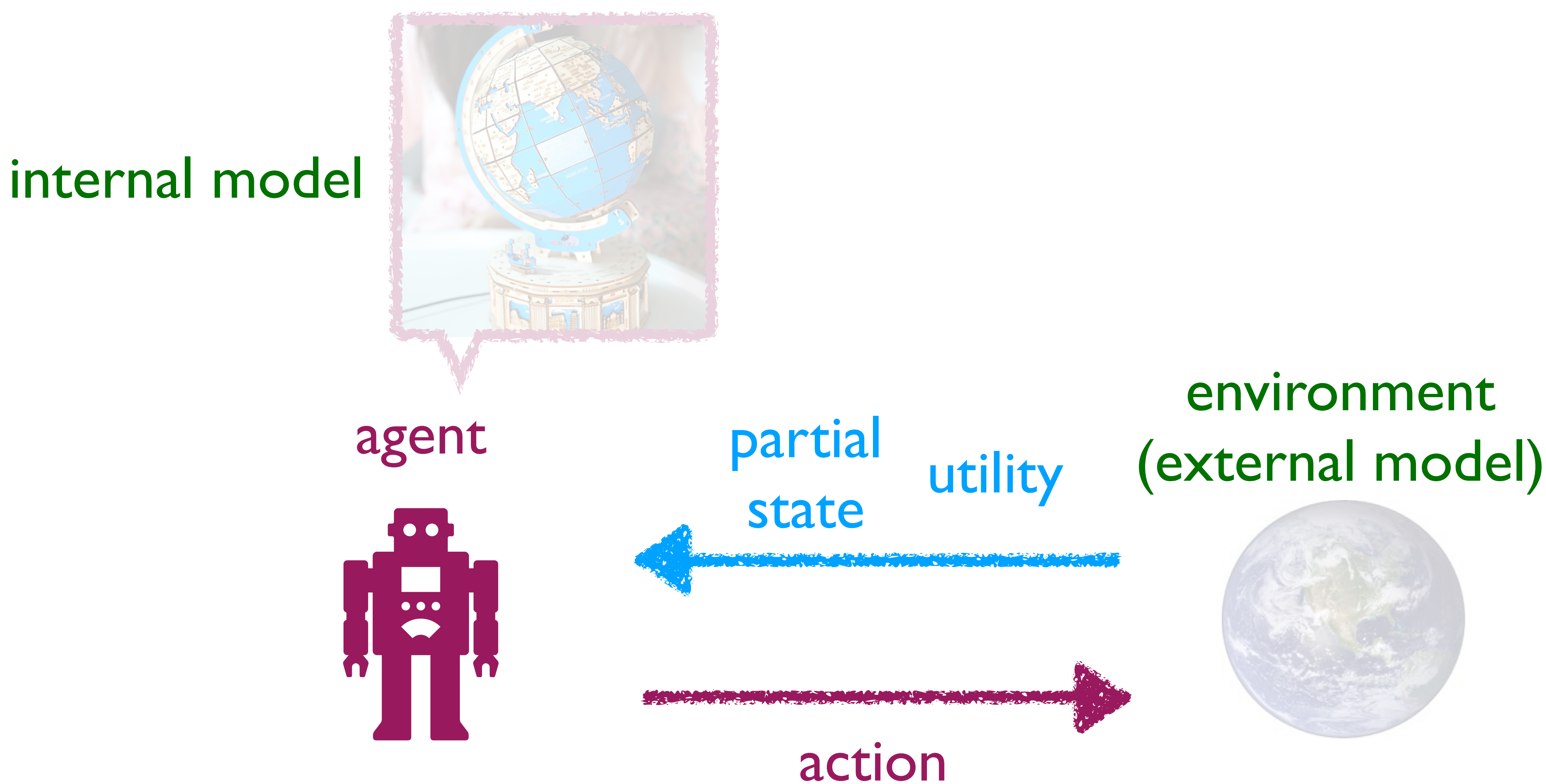
A decision-making agent can make use of external model when available, or build its own internal model when unavailable.



Internal vs. External Model

A decision-making agent can make use of external model when available, or build its own internal model when unavailable.

But what if both the external and internal models can not be used?



Reinforcement Learning

- Decision making is to find the optimal policy:
    - Decide best actions on all states.
    - No labeled <state, action> data, only receive reward.
    - The target is to maximize the long term total reward.

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- Search-based decision making:
  - When the model is known, and the search cost is reasonable.
- Reinforcement learning:
  - Decision making in unknown model or search cost is too high.

Reinforcement Learning

- Basics
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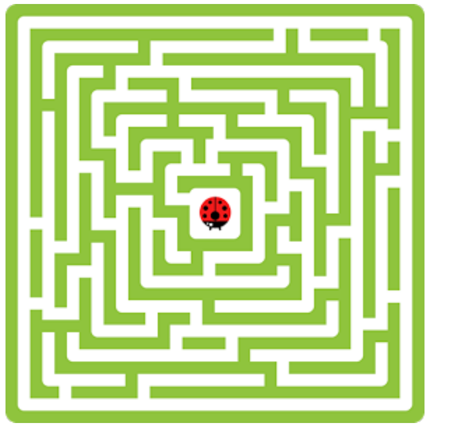
Markov Decision Process

- How to model a maze problem?



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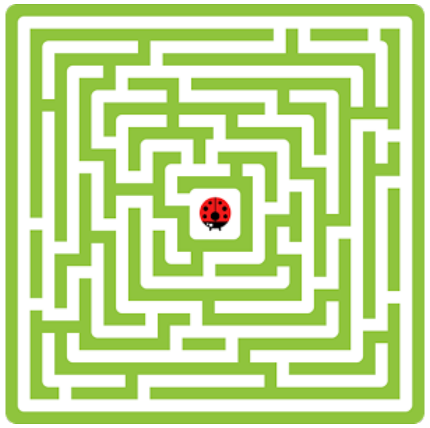

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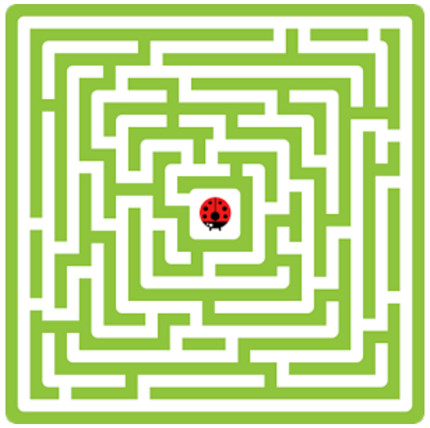

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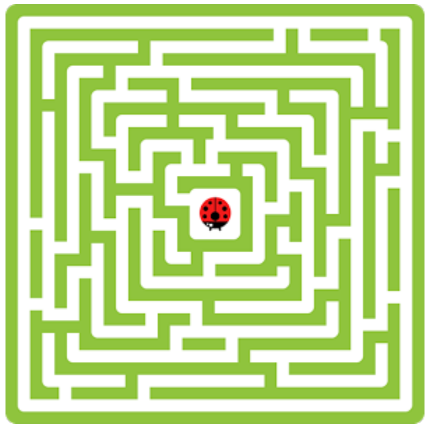

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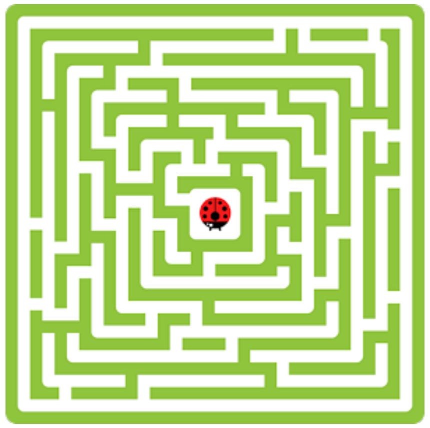

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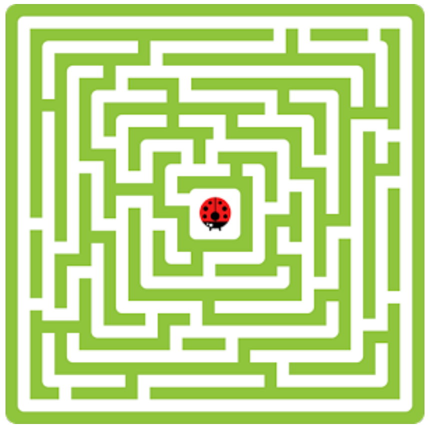

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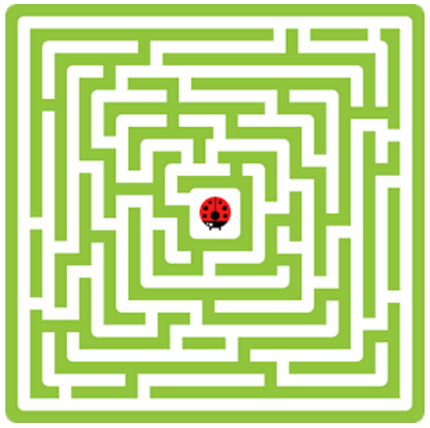

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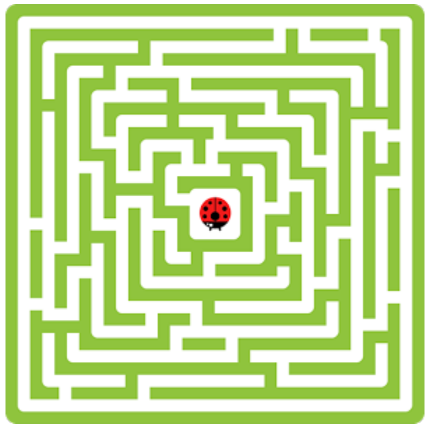
Markov decision process (MDP) is the decision making model in RL with specific assumptions.

Mathematical Formulation

- A Markov Decision Process (MDP) is a five-tuple



$$< \mathcal{S}, \mathcal{A}, \mathcal{P}, \mathcal{R}, \gamma >$$

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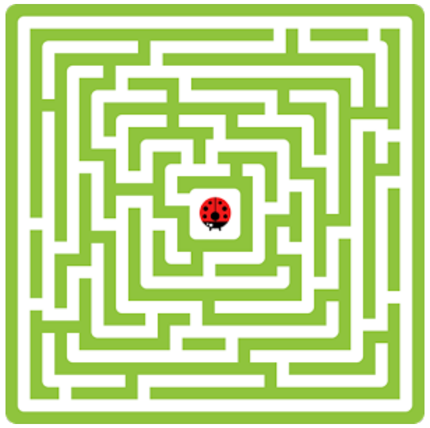

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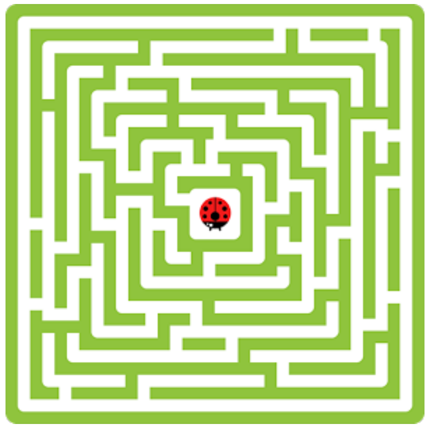

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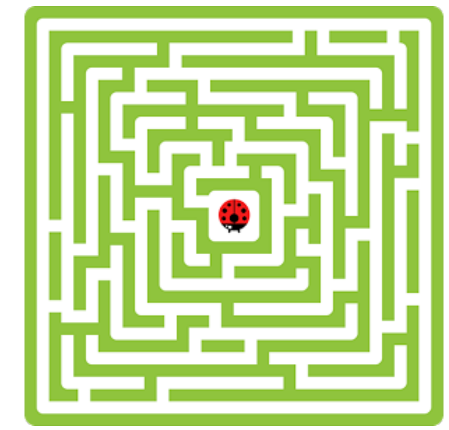

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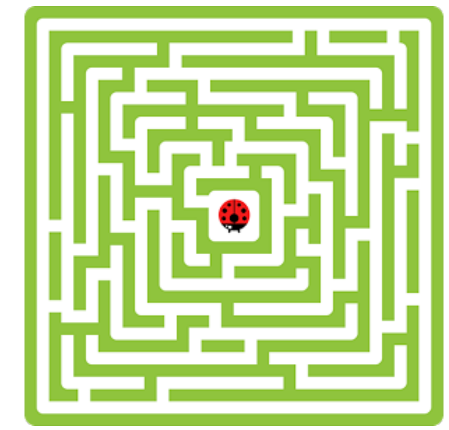

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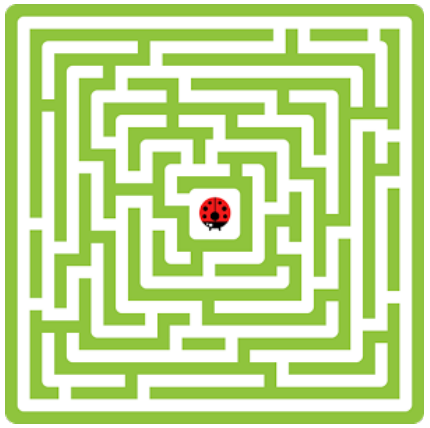

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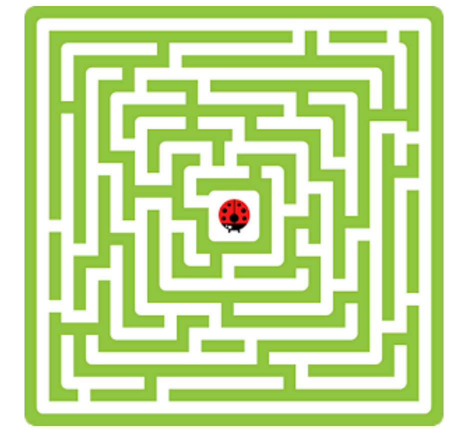

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The transition and reward functions can be stochastic!

The Markov Property

"The future is independent of the past given the present"

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- The current state is a sufficient statistic.
- Non-Markovian decision problem:

小张出生于中国，2016年来到美国留学。小张的母语是(？)

The Learning Agent

- The agent takes a series of actions, experiences a series of states, and receives a series of rewards:

$$\{s_1, a_1, r_1\}, \{s_2, a_2, r_2\}, \{s_3, a_3, r_3\}...$$

- policy: function $p(a|s)$ used to select actions on any states.

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- policy: function $p(a|s)$ used to select actions on any states.

- The target is to find the optimal policy to maximize the discounted total reward along the timeline:

$$r_1 + \gamma r_2 + \gamma^2 r_3 + ... = \sum_{t=1}^{\infty} \gamma^{t-1} r_t$$

- The discount factor measures how much the current action cares about the long term effect.

Value Functions: State Value Function V

- The state value function of a given policy is the expected total reward start from a given state, then follow the policy:

$$V_\pi(s) = \mathbb{E}_{\pi, p(s|s,a)}\Big[\sum_{t=0}^{\infty} \gamma^t r_t | s_0 = s\Big]$$

- The optimal policy have the optimal value function:

$$\forall s, \quad V_{\pi^*}(s) = \max_\pi V_\pi(s)$$

Value Functions: Action-State Value Function Q

- The action-state value function of a given policy is the expected total reward start from a given state, execute a given action, then follow the policy:

$$Q_\pi(s,a) = \mathbb{E}_{\pi,p(s|s,a)}\Big[\sum_{t=0}^{\infty} \gamma^t r_t | s_0 = s, a_0 = a\Big]$$

- The optimal deterministic policy chooses the optimal action:

$$\pi^*(s) = \arg\max_a Q_{\pi^*}(s,a)$$

If the optimal action-state value function is known, so is the optimal policy!

Bellman Equation

- For the state value function,

$$V_\pi(s) = \mathbb{E}_{\pi(s),p(s|s,a)}\Big[\sum_{t=0}^{\infty}\gamma^t r_t | s_0 = s\Big]$$

$$= \mathbb{E}_{\pi(s_0),p(s_1|s_0,a_0)}\Big[r_0 + \gamma \mathbb{E}_{\pi(s),p(s|s,a)}\Big[\sum_{t=1}^{\infty}\gamma^{t-1} r_t | s_1\Big] | s_0 = s\Big]$$

$$= \mathbb{E}_{\pi(s_0),p(s_1|s_0,a_0)}\Big[r_0 + \gamma \underline{V_\pi(s_1)} | s_0 = s\Big]$$

Recursive Definition

- For discrete state and action, and deterministic policy,

$$V_\pi(s) = \sum_{s'} p\big(s'|s,\pi(s)\big)\Big[r(s,\pi(s),s') + \gamma V_\pi(s')\Big]$$

Bellman Equation (Cont.)

- For the action-state value function,

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Bellman Equation (Cont.)

- For optimal deterministic policy π^*,

$$V_{\pi^*}(s) = \max_a \Big[\sum_{s'} p\big(s'|s,a\big)[r(s,a,s') + \gamma V_{\pi^*}(s')]\Big]$$

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- Then the optimal deterministic policy is

$$\pi^*(s) = \arg\max_a Q_{\pi^*}(s,a)$$

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- Then the optimal deterministic policy is

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- Due to the recursive structure, the optimal value functions can be solved by dynamical programming. This assumes that the full information of the MDP is known!

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Value Iteration

- Initialize value function V_0
- For i=1,2,3… until convergence
  - Update V_i for each state

$$V_i(s) = \max_a \Big[\sum_{s'} p\big(s'|s,a\big)[r(s,a,s') + \gamma V_{i-1}(s')]\Big]$$

- Theoretical convergence guarantee to V^* and π^*

Value Iteration

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Why iterative update?
Loop exists in the MDP!

$$V_i(s) = \max_a \Big[\sum_{s'} p(s'|s,a) [r(s,a,s') + \gamma V_{i-1}(s')] \Big]$$

- Theoretical convergence guarantee to V^* and π^*

Policy Iteration

- Initialize value function V_0 and policy π_0
- For i=1,2,3… until convergence
  - Policy evaluation step: update V_i for each state until converge

$$V_i(s) = r + \gamma V_{i-1}^{\pi_{i-1}}(s')$$

  - Policy improvement step: update π_i for each s-a pair.

$$\pi(s) \leftarrow arg\,\max_a \sum_{s',r'} p(s', r|s, a)[r + \gamma V(s')]$$

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Policy Iteration

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  - Policy evaluation step: update V_i for each state until converge

$$V_i(s) = r + \gamma V_{i-1}^{\pi_{i-1}}(s')$$

Calculated based on π_{i-1}
Actually an inner loop to do iterative update until convergence

  - Policy improvement step: update π_i for each s-a pair.

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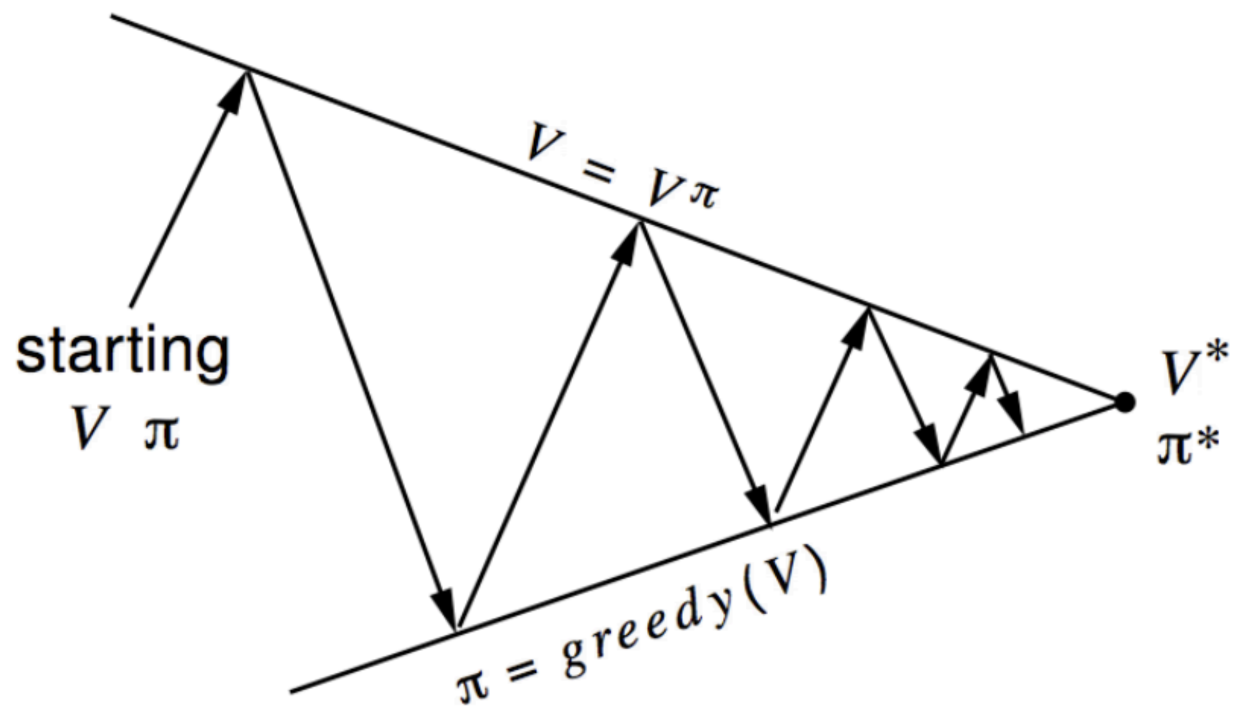
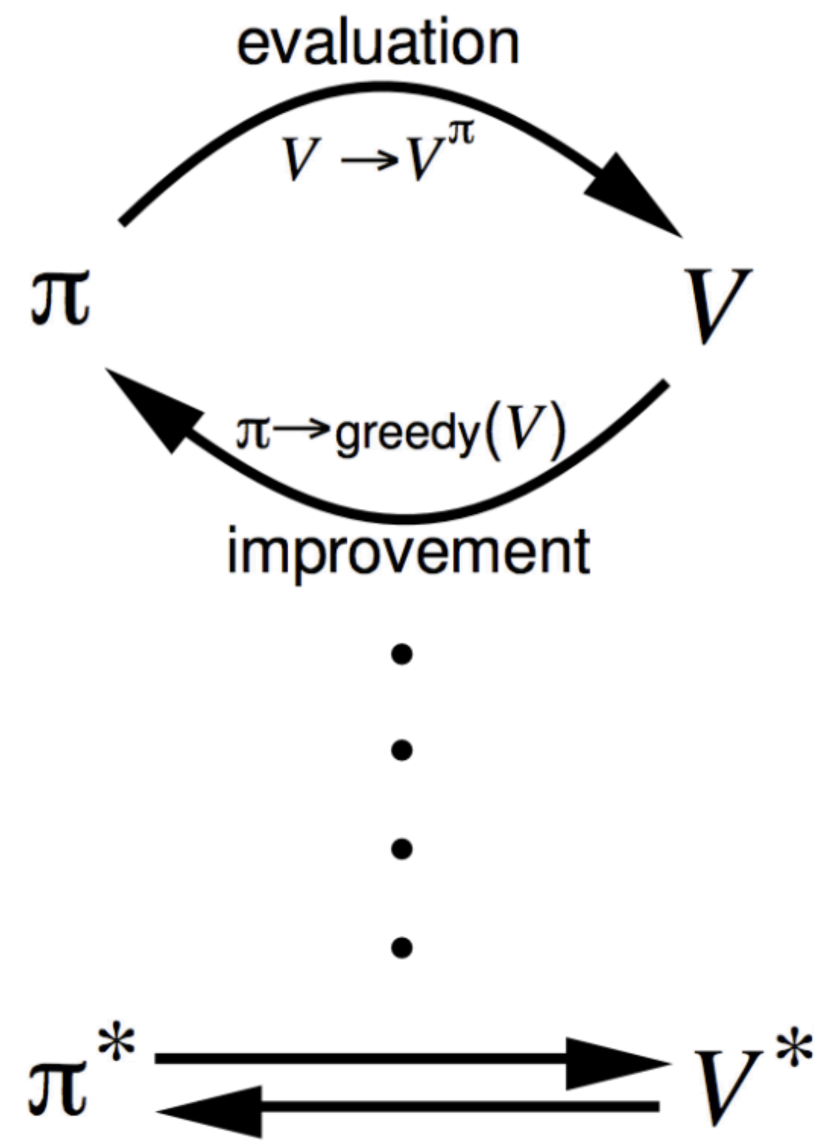
Policy Iteration (Cont.)





Policy evaluation Estimate v_π

Iterative policy evaluation

Policy improvement Generate $\pi' \geq \pi$

Greedy policy improvement

Slide courtesy: David Silver

Learning with Unknown Model

- When full information of the MDP is known, the value function can be solved by planning.
- But how to solve when not fully known? $< \mathcal{S}, \mathcal{A}, \underline{\mathcal{P}, \mathcal{R},} \gamma >$
- In RL, usually the state transition $\mathcal{P}$ and reward function $\mathcal{R}$ are not known.
- The agent has to learn by trial and error, facing with the exploration and exploitation problem.

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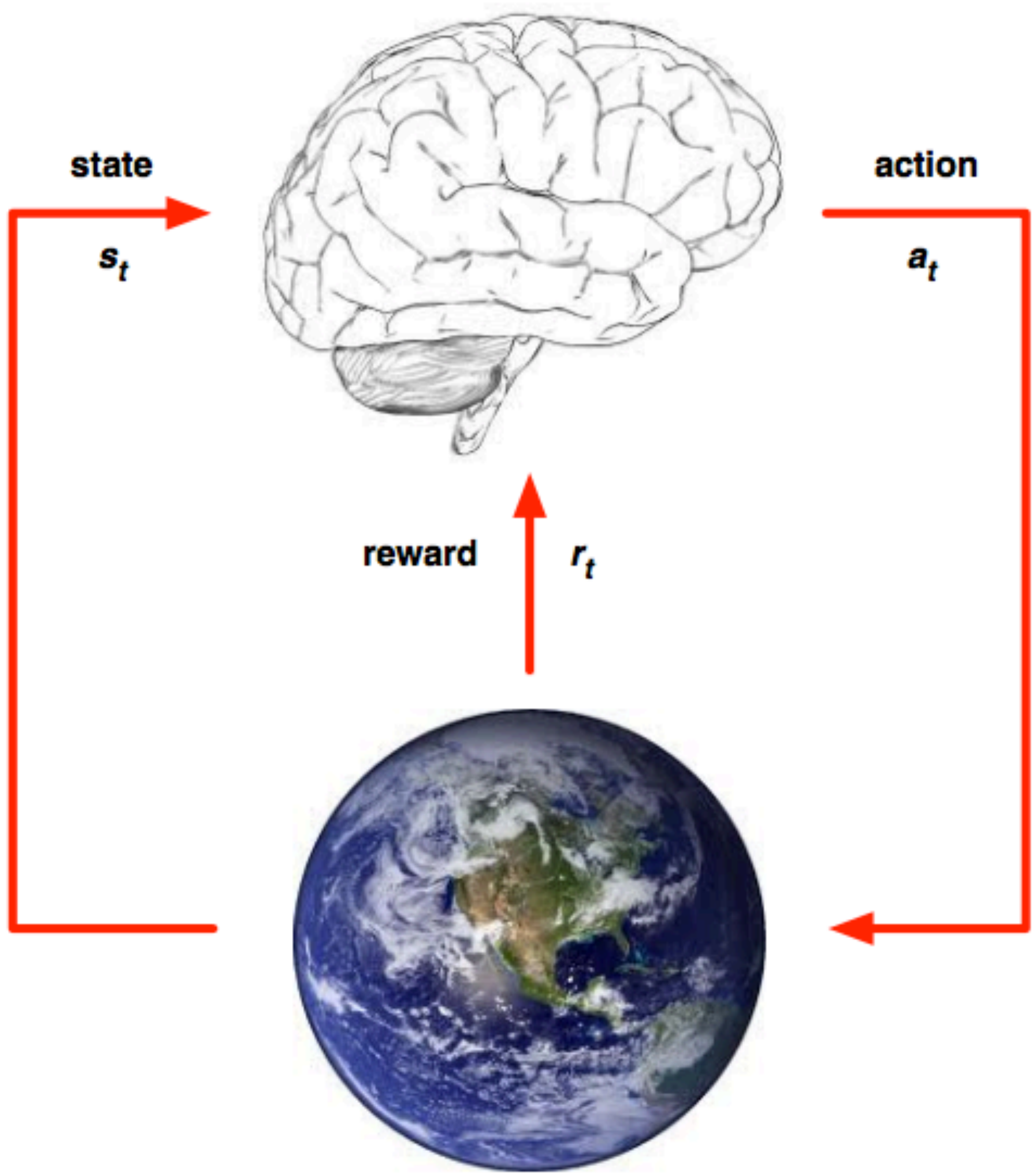
Agent and Environment



- At each step t the agent:
  - Receives state s_t
  - Receives scalar reward r_t
  - Executes action a_t
- The environment:
  - Receives action a_t
  - Emits state s_t
  - Emits scalar reward r_t

- The target is still to learn the optimal value function.

Slide courtesy: David Silver

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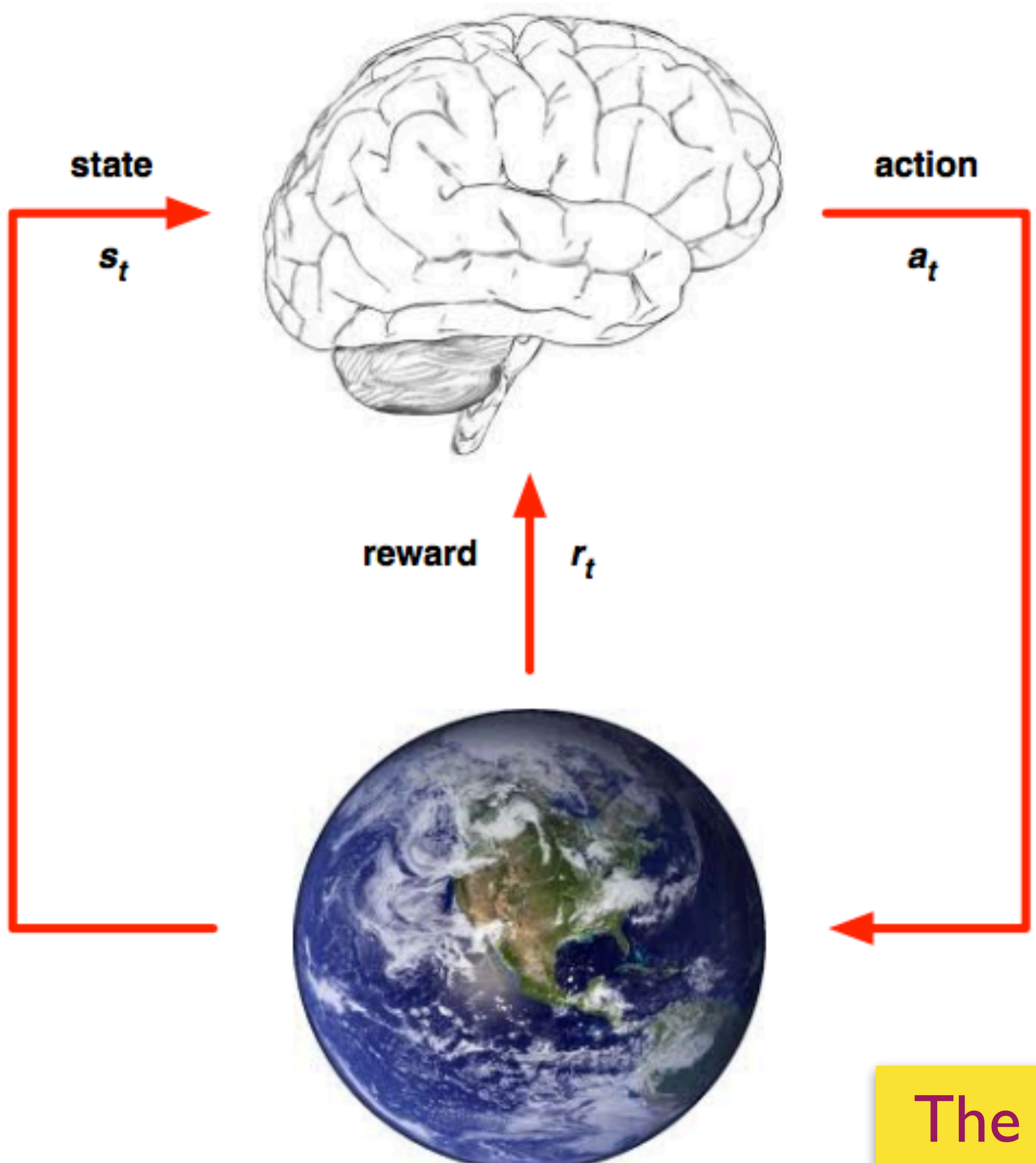

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The agent can only interact with true environment.
Can not use model for search or planning.

- The target is still to learn the optimal value function.

Slide courtesy: David Silver

Basic Idea

- Similar to DP, aiming at estimating the optimal value function.

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- Value function update: update using new estimation

$$Q_{i+1}(s,a) = (1-\alpha)Q_i(s,a) + \alpha\hat{Q}_i(s,a)$$

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  - Temporal difference Learning — SARSA and Q-learning.

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  - Monte-Carlo RL — Estimate by sampled trajectories
  - Temporal difference Learning — SARSA and Q-learning.
- Policy Improvement:
  - Based on new value function, with ϵ - greedy.

Monte-Carlo RL

- Given policy π_i, we can sample trajectories:

$$\{s_1, a_1, r_1\}, \{s_2, a_2, r_2\}, \{s_3, a_3, r_3\} ...$$

- Then we can get empirical estimate:

$$\hat{Q}_i(s_1, a_1) = r_1 + \gamma r_2 + \gamma^2 r_3 + ...$$

- Update value function:

$$Q_{i+1}(s, a) = (1 - \alpha) Q_i(s, a) + \alpha \hat{Q}_i(s, a)$$

- Follow the spirit of policy iteration, update $\pi_i \rightarrow \pi_{i+1}$

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Can we still update the policy greedily?

- Update value function:

$$Q_{i+1}(s, a) = (1 - \alpha)Q_i(s, a) + \alpha\hat{Q}_i(s, a)$$

No!

- Follow the spirit of policy iteration, update $\pi_i \to \pi_{i+1}$

Exploration vs. Exploitation


"Behind one door is tenure - behind the other is flipping burgers at McDonald's."

- There are two doors in front of you.
- You open the left door and get reward 0
  $V(left) = 0$
- You open the right door and get reward +1
  $V(right) = +1$
- You open the right door and get reward +3
  $V(right) = +2$
- You open the right door and get reward +2
  $V(right) = +2$

$\vdots$

- Are you sure you've chosen the best door?

Slide courtesy: David Silver

Exploration vs. Exploitation (Cont.)

- In policy iteration, the policy improvement step is greedy:

$$\pi_i(s) = \arg\max_a Q_i(s, a)$$

- But for RL, since the environment is not fully known, greedy update may perform arbitrarily bad — need to allow some exploration.

- Common choice: use ϵ- greedy policy:

  with prob. $1 - \epsilon$, execute as greedy

  with prob. ϵ, execute randomly

- Theoretical guarantee: If the exploration vanishes, we can ensure convergence.

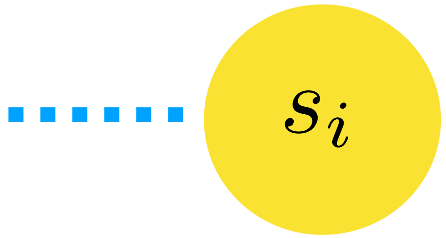
TD vs. MC

- Temporal-difference (TD) learning has several advantages over Monte-Carlo (MC)
  - Lower variance
  - Online
  - Incomplete sequences
- Natural idea: use TD instead of MC in our control loop
  - Apply TD to $Q(S, A)$
  - Use ϵ-greedy policy improvement
  - Update every time-step

Slide courtesy: David Silver

SARSA

- “State-Action-Reward-State-Action” — SARSA



Run the current
best ϵ-greedy policy

SARSA

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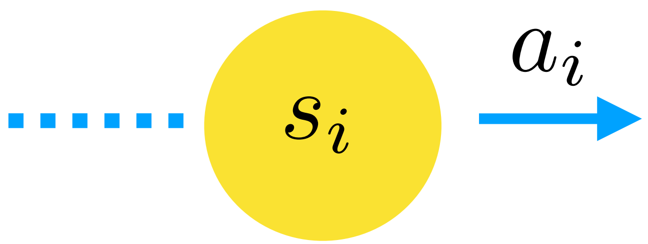

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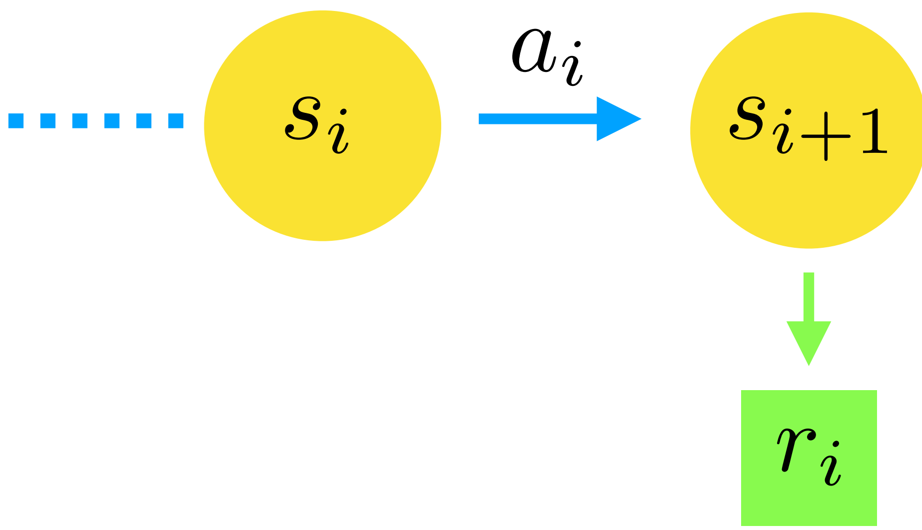

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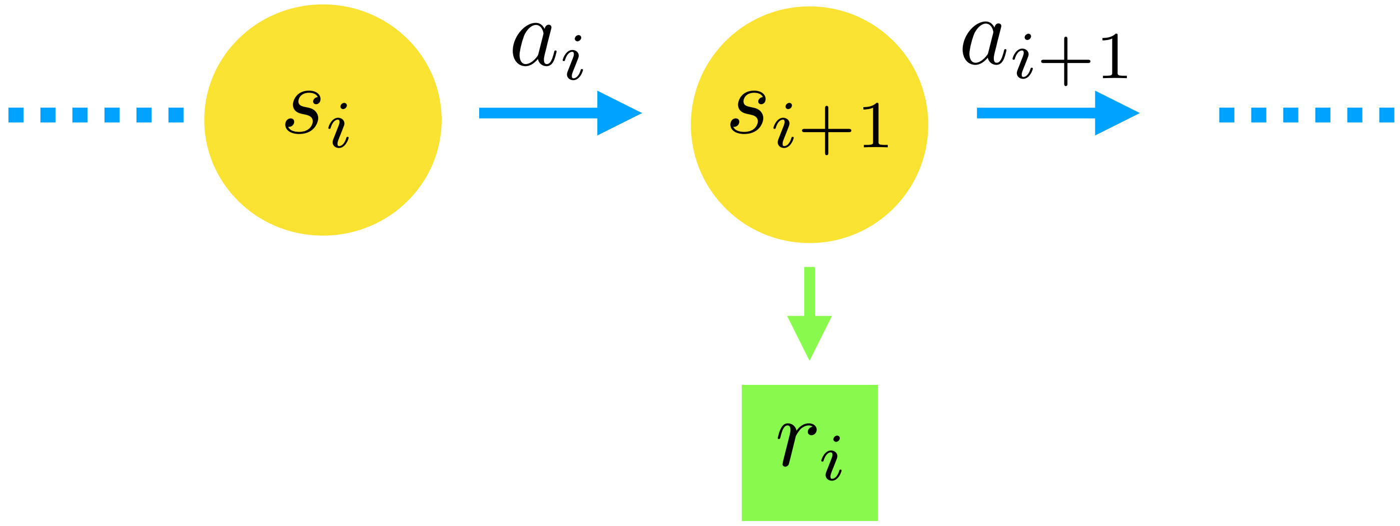

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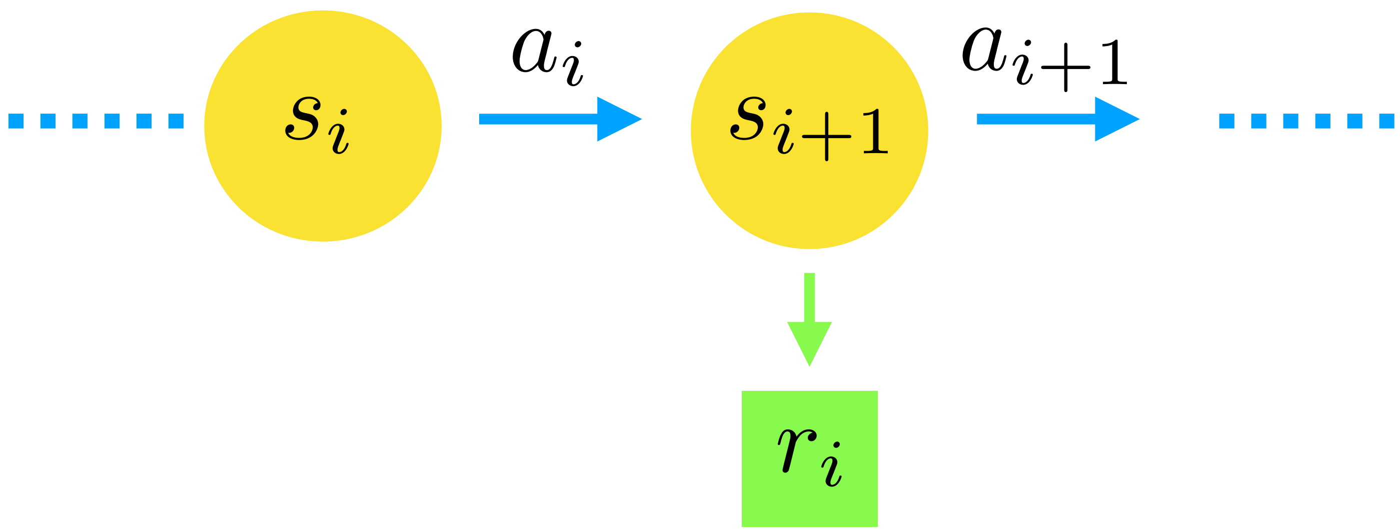

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- Once collect s-a-r-s-a sample, do value function update:

$$Q(s_i, a_i) \leftarrow Q(s_i, a_i) + \alpha \left[r_i + \gamma Q(s_{i+1}, a_{i+1}) - Q(s_i, a_i) \right]$$

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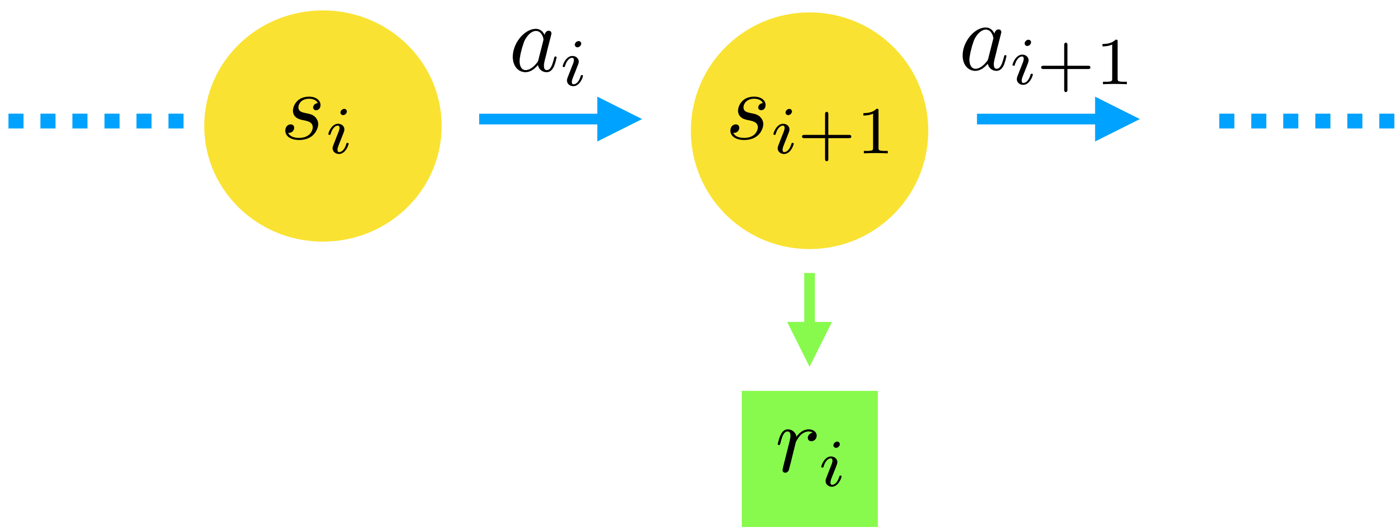

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TD error

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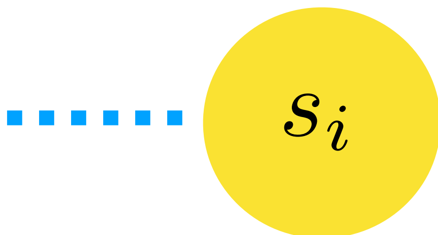
Always use the policy on-the-run, called "on-policy"

Q-Learning



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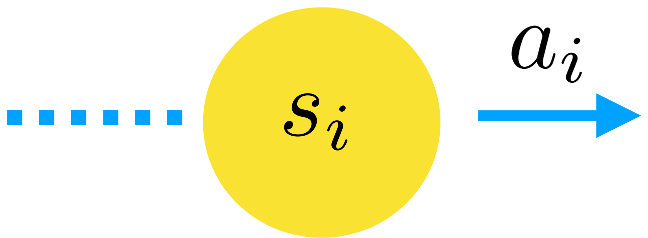

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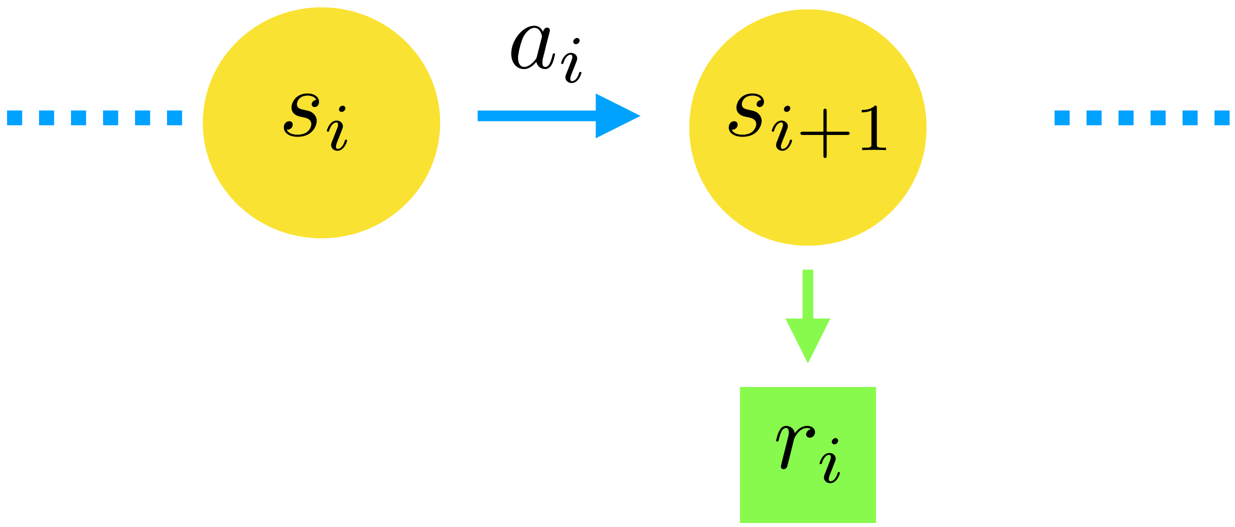

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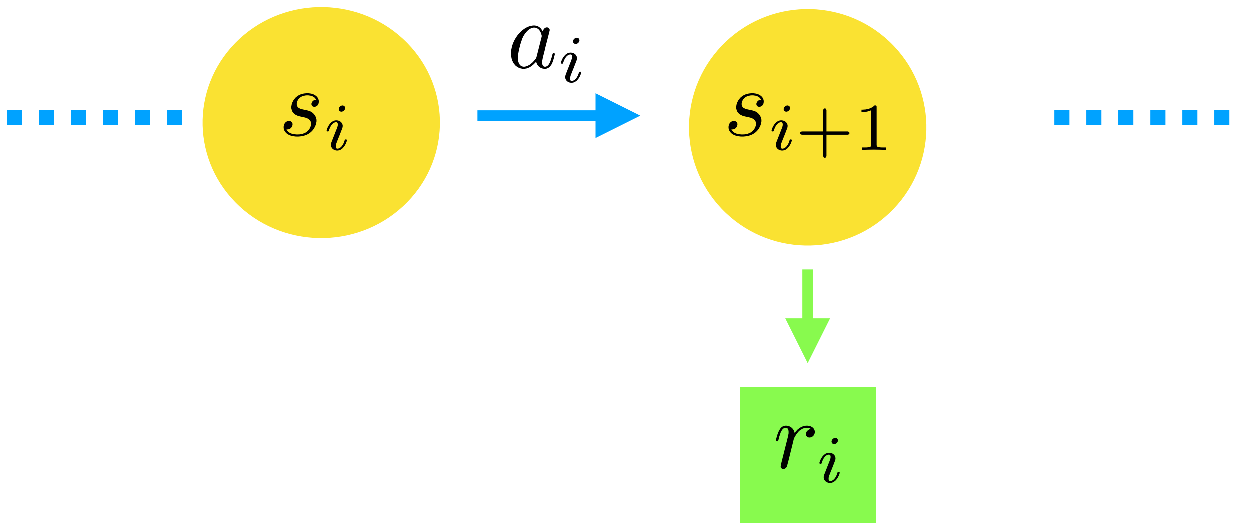

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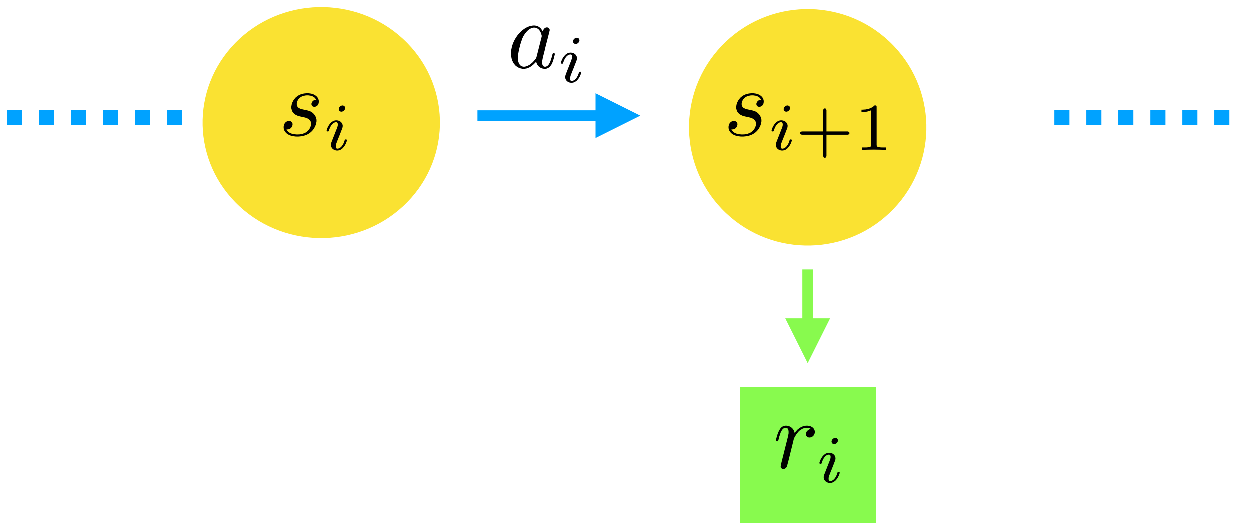

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The current best policy

The policy on the run can be different from the current best policy in the update, called "off-policy"

Off-Policy Learning

- Evaluate target policy $\pi(a|s)$ to compute $v_\pi(s)$ or $q_\pi(s,a)$
- While following behaviour policy $\mu(a|s)$

$$\{S_1, A_1, R_2, ..., S_T\} \sim \mu$$

- Why is this important?
- Learn from observing humans or other agents
- Re-use experience generated from old policies $\pi_1, \pi_2, ..., \pi_{t-1}$
- Learn about *optimal* policy while following *exploratory* policy
- Learn about *multiple* policies while following *one* policy

Slide courtesy: David Silver

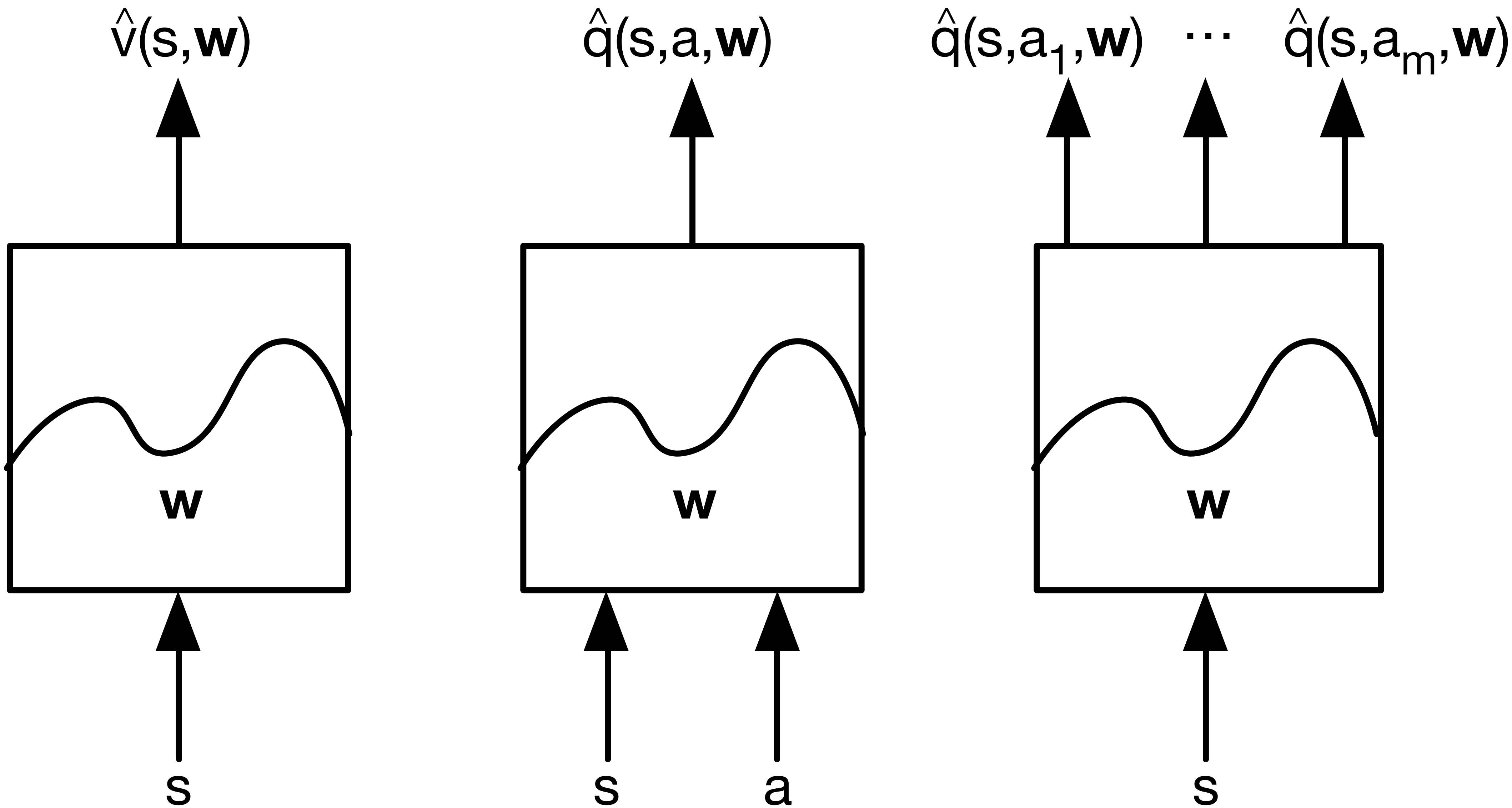
Large-Scale RL

- Large decision-making problems:
  - Backgammon: 10^{20} states
  - Go: 10^{170} states
  - Robot control: continuous state space

Classic value function methods rely on tabular representation of value functions.
Obviously needing more compact representations.

Types of Value Function Approximation

output: value function scores



input: state or/and actions

Slide courtesy: David Silver

Feature Vectors

- Represent state by a *feature vector*

$$\mathbf{x}(S) = \begin{pmatrix} \mathbf{x}_1(S) \\ \vdots \\ \mathbf{x}_n(S) \end{pmatrix}$$

- For example:
  - Distance of robot from landmarks
  - Trends in the stock market
  - Piece and pawn configurations in chess

Slide courtesy: David Silver

Linear Value Function Approximation

- Represent value function by a linear combination of features

$$\hat{v}(S, \mathbf{w}) = \mathbf{x}(S)^\top \mathbf{w} = \sum_{j=1}^{n} \mathbf{x}_j(S) \mathbf{w}_j$$

- Objective function is quadratic in parameters $\mathbf{w}$

$$J(\mathbf{w}) = \mathbb{E}_\pi \left[(v_\pi(S) - \mathbf{x}(S)^\top \mathbf{w})^2 \right]$$

- Stochastic gradient descent converges on *global* optimum
- Update rule is particularly simple

$$\nabla_{\mathbf{w}} \hat{v}(S, \mathbf{w}) = \mathbf{x}(S)$$

$$\Delta \mathbf{w} = \alpha (v_\pi(S) - \hat{v}(S, \mathbf{w})) \mathbf{x}(S)$$

Update $=$ *step-size* $\times$ *prediction error* $\times$ *feature value*

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beyond simple linear regression

Function Approximators

There are many function approximators, e.g.

- Linear combinations of features
- Neural network
- Decision tree
- Nearest neighbour
- Fourier / wavelet bases
- ...

Slide courtesy: David Silver

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more commonly used

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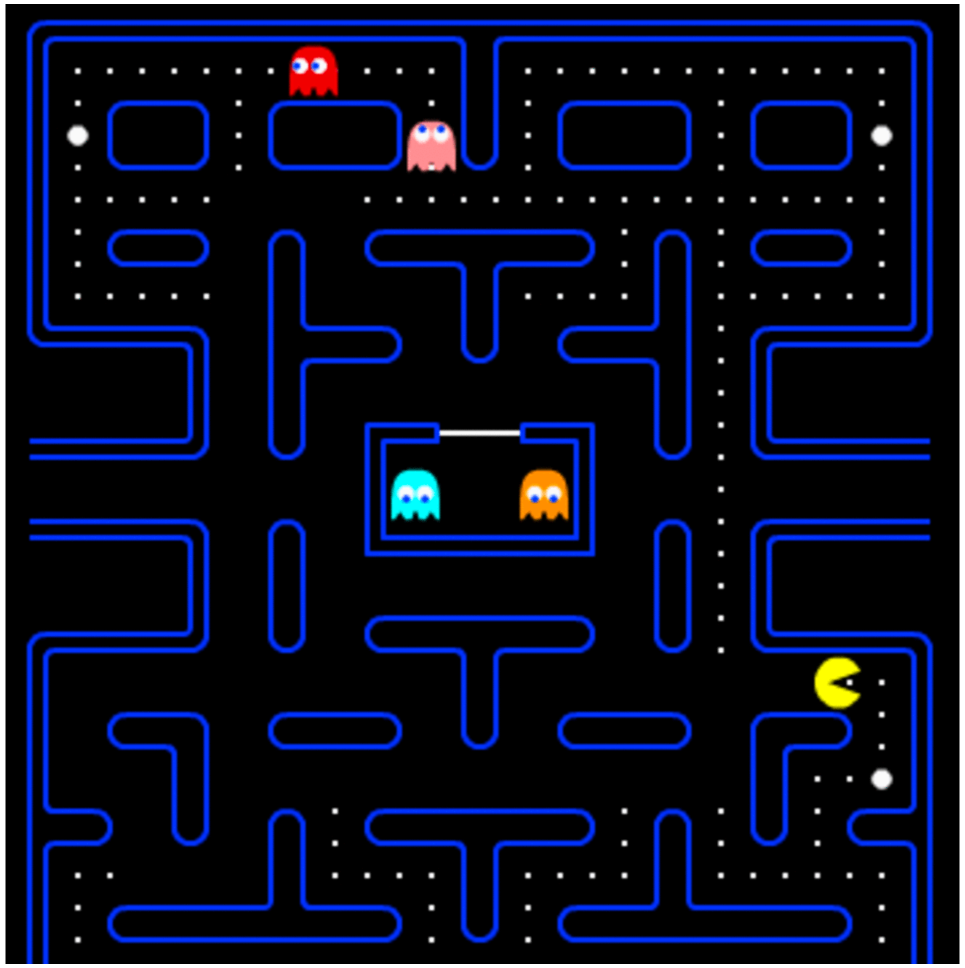
more commonly used

Different from traditional supervised learning,
we need learning algorithms that can handle data collected by the learner online:
biased and unstable.

Slide courtesy: David Silver

Bias and Instability



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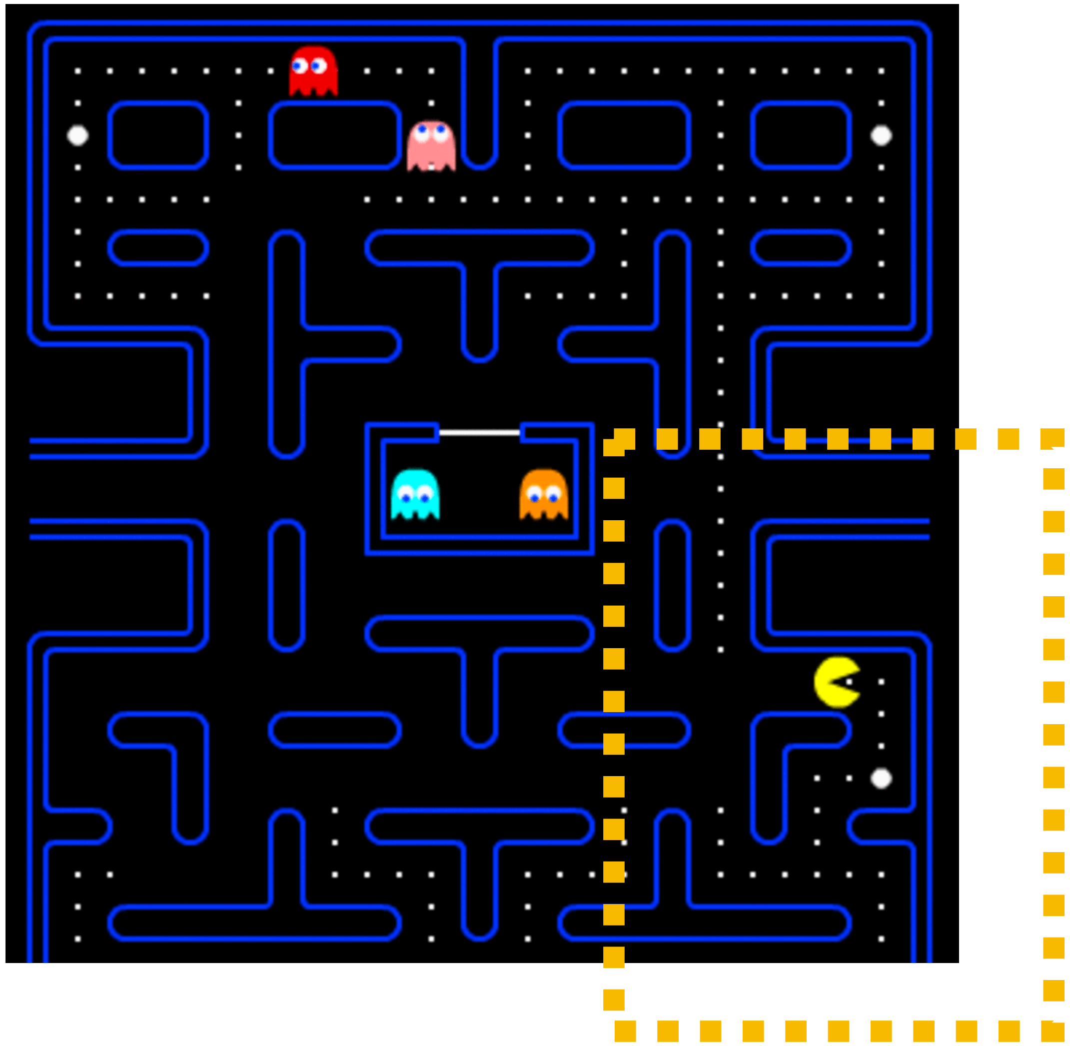

The learner collects data by itself. “Only see what it want to see”

The issue of bias and instability for data collection lies at the heart of RL.
This is also why we need exploration.

Approximate Targets via Bellman Equation

- Have assumed true value function $v_\pi(s)$ given by supervisor
- But in RL there is no supervisor, only rewards

- In practice, we substitute a *target* for $v_\pi(s)$
  - For MC, the target is the return G_t

$$\Delta \mathbf{w} = \alpha(G_t - \hat{v}(S_t, \mathbf{w}))\nabla_{\mathbf{w}}\hat{v}(S_t, \mathbf{w})$$

  - For TD(0), the target is the TD target $R_{t+1} + \gamma \hat{v}(S_{t+1}, \mathbf{w})$

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  - For TD(λ), the target is the λ-return G_t^λ

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Monte-Carlo estimation

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Temporal-difference estimation

Slide courtesy: David Silver

Action Value Function Approximation

- Approximate the action-value function

$$\hat{q}(S, A, \mathbf{w}) \approx q_\pi(S, A)$$

- Minimise mean-squared error between approximate action-value fn $\hat{q}(S, A, \mathbf{w})$ and true action-value fn $q_\pi(S, A)$

$$J(\mathbf{w}) = \mathbb{E}_\pi \left[(q_\pi(S, A) - \hat{q}(S, A, \mathbf{w}))^2 \right]$$

- Use stochastic gradient descent to find a local minimum

$$-\frac{1}{2}\nabla_{\mathbf{w}} J(\mathbf{w}) = (q_\pi(S, A) - \hat{q}(S, A, \mathbf{w}))\nabla_{\mathbf{w}}\hat{q}(S, A, \mathbf{w})$$

$$\Delta\mathbf{w} = \alpha(q_\pi(S, A) - \hat{q}(S, A, \mathbf{w}))\nabla_{\mathbf{w}}\hat{q}(S, A, \mathbf{w})$$

Slide courtesy: David Silver

Linear Action-Value Function Approximation

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$$\mathbf{x}(S,A) = \begin{pmatrix} \mathbf{x}_1(S,A) \\ \vdots \\ \mathbf{x}_n(S,A) \end{pmatrix}$$

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- Stochastic gradient descent update

$$\begin{aligned} \nabla_{\mathbf{w}} \hat{q}(S,A,\mathbf{w}) &= \mathbf{x}(S,A) \\ \Delta \mathbf{w} &= \alpha(q_\pi(S,A) - \hat{q}(S,A,\mathbf{w}))\mathbf{x}(S,A) \end{aligned}$$

Slide courtesy: David Silver

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E_t &= \gamma\lambda E_{t-1} + \nabla_{\mathbf{w}}\hat{q}(S_t, A_t, \mathbf{w}) \\
\Delta \mathbf{w} &= \alpha \delta_t E_t
\end{aligned}$$

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SARSA here. Can also do Q-learning (more later)

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Slide courtesy: David Silver

Reinforcement Learning

- Basics
  - Markov decision process
  - RL with known model
  - RL with unknown model
  - Policy gradient & actor-critic methods
- Deep reinforcement learning
- Integrating learning and planning
- RL from human preference
- Take-home messages

Slides link:

Direct Policy Learning

- For value function based RL, policy is not directly optimized.
  - Not capable to learn in continuous state and action space.
  - Not capable to learn stochastic policy.
  - May not learn fast.

Direct Policy Learning

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    - Not capable to learn in continuous state and action space.
    - Not capable to learn stochastic policy.
    - May not learn fast.

- Can learn stochastic policy directly:
    - Parametrize policy $\pi_\theta(a|s)$ with parameter θ
        - For discrete action: softmax $\pi_\theta(s, a) \propto e^{\phi(s,a)^\top \theta}$
        - For continuous action: Gaussian $a \sim \mathcal{N}(\mu(s), \sigma^2)$

Direct Policy Learning

Advantages:

- Better convergence properties
- Effective in high-dimensional or continuous action spaces
- Can learn stochastic policies

Disadvantages:

- Typically converge to a local rather than global optimum
- Evaluating a policy is typically inefficient and high variance

Slide courtesy: David Silver

Objective Function

- Goal: given policy $\pi_\theta(s, a)$ with parameters θ, find best θ
- But how do we measure the quality of a policy π_θ?
- In episodic environments we can use the start value

$$J_1(\theta) = V^{\pi_\theta}(s_1) = \mathbb{E}_{\pi_\theta}[v_1]$$

- In continuing environments we can use the average value

$$J_{avV}(\theta) = \sum_s d^{\pi_\theta}(s) V^{\pi_\theta}(s)$$

- Or the average reward per time-step

$$J_{avR}(\theta) = \sum_s d^{\pi_\theta}(s) \sum_a \pi_\theta(s, a) \mathcal{R}_s^a$$

- where $d^{\pi_\theta}(s)$ is stationary distribution of Markov chain for π_θ

Slide courtesy: David Silver

Objective Function

- Goal: given policy $\pi_\theta(s, a)$ with parameters θ, find best θ
- But how do we measure the quality of a policy π_θ?
- In episodic environments we can use the start value

$$J_1(\theta) = V^{\pi_\theta}(s_1) = \mathbb{E}_{\pi_\theta}[v_1]$$

- In continuing environments we can use the average value

$$J_{avV}(\theta) = \sum_s d^{\pi_\theta}(s) V^{\pi_\theta}(s)$$

- Or the average reward per time-step

$$J_{avR}(\theta) = \sum_s d^{\pi_\theta}(s) \sum_a \pi_\theta(s, a) \mathcal{R}_s^a$$

- where $d^{\pi_\theta}(s)$ is stationary distribution of Markov chain for π_θ

The challenge is that the distribution can only be estimated when the agent itself samples data.

Slide courtesy: David Silver

Policy Gradient

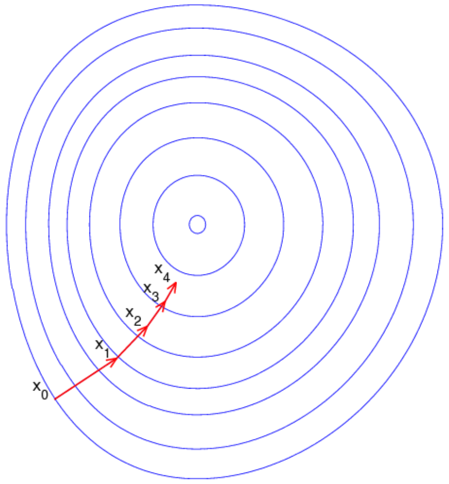
- Let $J(\theta)$ be any policy objective function
- Policy gradient algorithms search for a *local* maximum in $J(\theta)$ by ascending the gradient of the policy, w.r.t. parameters θ

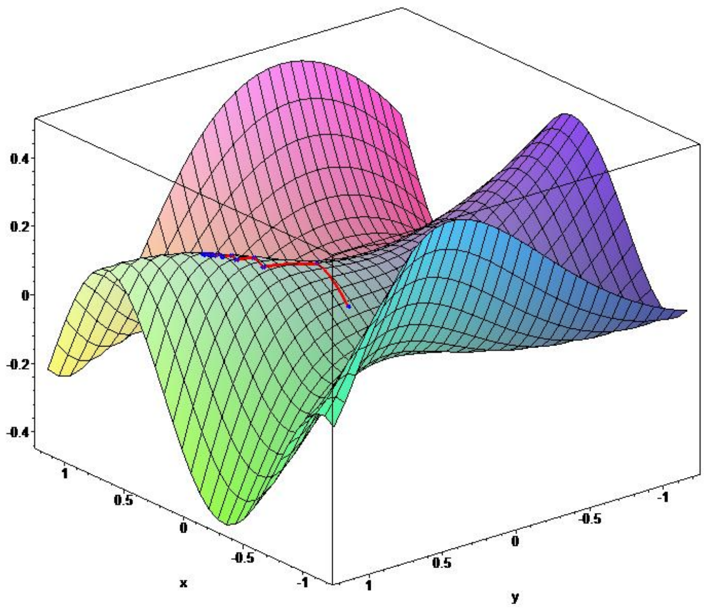
$$\Delta\theta = \alpha \nabla_\theta J(\theta)$$

- Where $\nabla_\theta J(\theta)$ is the policy gradient

$$\nabla_\theta J(\theta) = \begin{pmatrix} \frac{\partial J(\theta)}{\partial \theta_1} \\ \vdots \\ \frac{\partial J(\theta)}{\partial \theta_n} \end{pmatrix}$$

- and α is a step-size parameter





Slide courtesy: David Silver

Policy Gradient Theorem

Policy Gradient Methods for Reinforcement Learning with Function Approximation

Richard S. Sutton, David McAllester, Satinder Singh, Yishay Mansour
AT&T Labs – Research, 180 Park Avenue, Florham Park, NJ 07932

Theorem

For any differentiable policy $\pi_\theta(s, a)$,
for any of the policy objective functions $J = J_1, J_{avR},$ *or* $\frac{1}{1-\gamma} J_{avV}$,
the policy gradient is

$$\nabla_\theta J(\theta) = \mathbb{E}_{\pi_\theta} \left[\nabla_\theta \log \pi_\theta(s, a) \; Q^{\pi_\theta}(s, a) \right]$$

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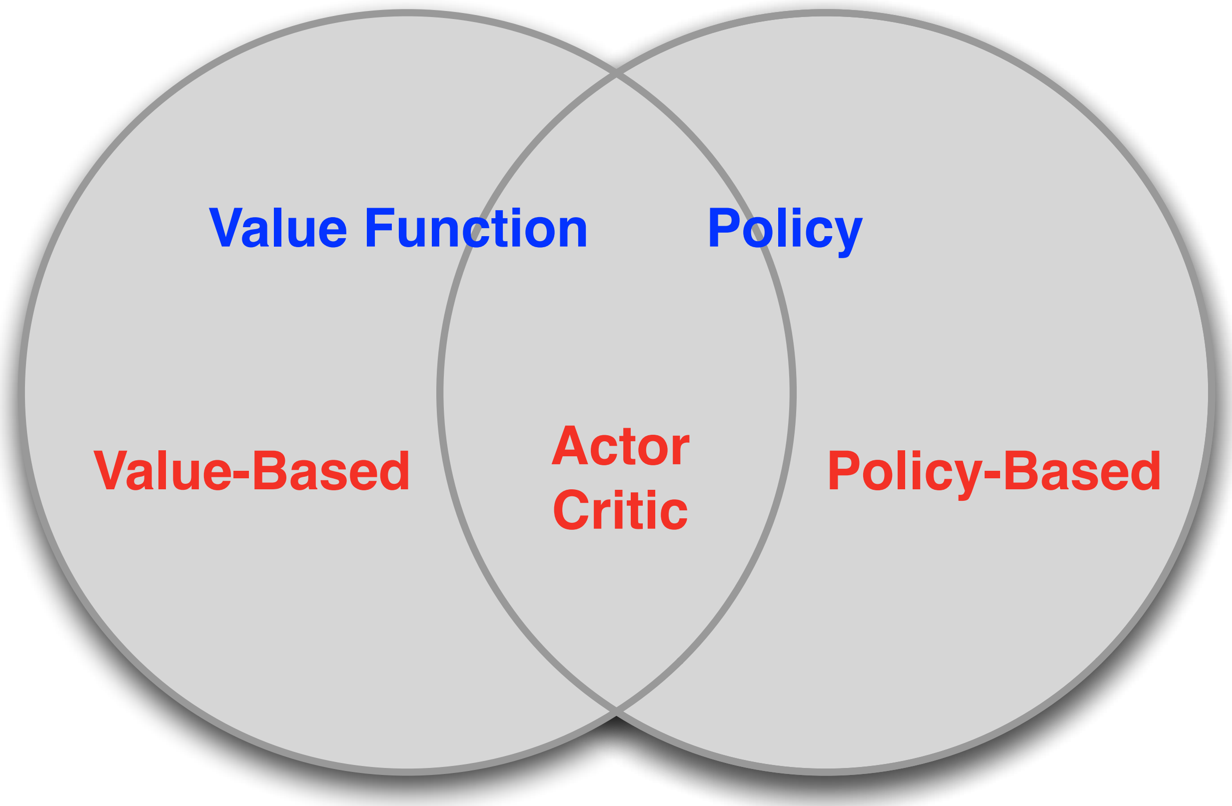
$$\nabla_\theta J(\theta) = \mathbb{E}_{\pi_\theta}\left[\nabla_\theta \log \pi_\theta(s, a)\, Q^{\pi_\theta}(s, a)\right]$$

Need to estimate value function

Monte-Carlo or temporal difference

Actor-Critic

- Value Based
  - Learnt Value Function
  - Implicit policy (e.g. ϵ-greedy)
- Policy Based
  - No Value Function
  - Learnt Policy
- Actor-Critic
  - Learnt Value Function
  - Learnt Policy



Slide courtesy: David Silver

Reinforcement Learning

- Basics
  - Markov decision process
  - RL with known model
  - RL with unknown model
  - Policy gradient & actor-critic methods
- Deep reinforcement learning
- Integrating learning and planning
- RL from human preference
- Take-home messages

Slides link:

Practical Issues for RL

- Reward design: an art
- State feature design: also an art
- The environment is too complicated to model.

  - Can we apply deep learning to RL?
  - Use deep network to represent value function / policy / model
  - Optimise value function / policy /model end-to-end
  - Using stochastic gradient descent

Slide courtesy: David Silver

Deep Q-Network

- Represent value function by deep Q-network with weights w

$$Q(s, a, w) \approx Q^{\pi}(s, a)$$

- Define objective function by mean-squared error in Q-values

$$\mathcal{L}(w) = \mathbb{E}\left[\left(\underbrace{r + \gamma \max_{a'} Q(s', a', w)}_{\text{target}} - Q(s, a, w)\right)^2\right]$$

- Leading to the following Q-learning gradient

$$\frac{\partial \mathcal{L}(w)}{\partial w} = \mathbb{E}\left[\left(r + \gamma \max_{a'} Q(s', a', w) - Q(s, a, w)\right) \frac{\partial Q(s, a, w)}{\partial w}\right]$$

- Optimise objective end-to-end by SGD, using $\frac{\partial L(w)}{\partial w}$

Recall function approximation in P. 42.

Slide courtesy: David Silver

Stability Issues for Deep RL

Naive Q-learning oscillates or diverges with neural nets

1. Data is sequential
   - Successive samples are correlated, non-iid
2. Policy changes rapidly with slight changes to Q-values
   - Policy may oscillate
   - Distribution of data can swing from one extreme to another
3. Scale of rewards and Q-values is unknown
   - Naive Q-learning gradients can be large unstable when backpropagated

The bias and instability issue we have discussed.
More severe for NNs.

Slide courtesy: David Silver

DQN Techs

DQN provides a stable solution to deep value-based RL

1. Use experience replay
   - Break correlations in data, bring us back to iid setting
   - Learn from all past policies
2. Freeze target Q-network
   - Avoid oscillations
   - Break correlations between Q-network and target
3. Clip rewards or normalize network adaptively to sensible range
   - Robust gradients

Experience Replay

To remove correlations, build data-set from agent's own experience

- Take action a_t according to ϵ-greedy policy
- Store transition $(s_t, a_t, r_{t+1}, s_{t+1})$ in replay memory $\mathcal{D}$
- Sample random mini-batch of transitions (s, a, r, s') from $\mathcal{D}$
- Optimise MSE between Q-network and Q-learning targets, e.g.

$$\mathcal{L}(w) = \mathbb{E}_{s,a,r,s' \sim \mathcal{D}} \left[\left(r + \gamma \max_{a'} Q(s', a', w) - Q(s, a, w) \right)^2 \right]$$

off-policy!

Fixed Network

To avoid oscillations, fix parameters used in Q-learning target

- Compute Q-learning targets w.r.t. old, fixed parameters w^-

$$r + \gamma \max_{a'} Q(s', a', w^-)$$

- Optimise MSE between Q-network and Q-learning targets

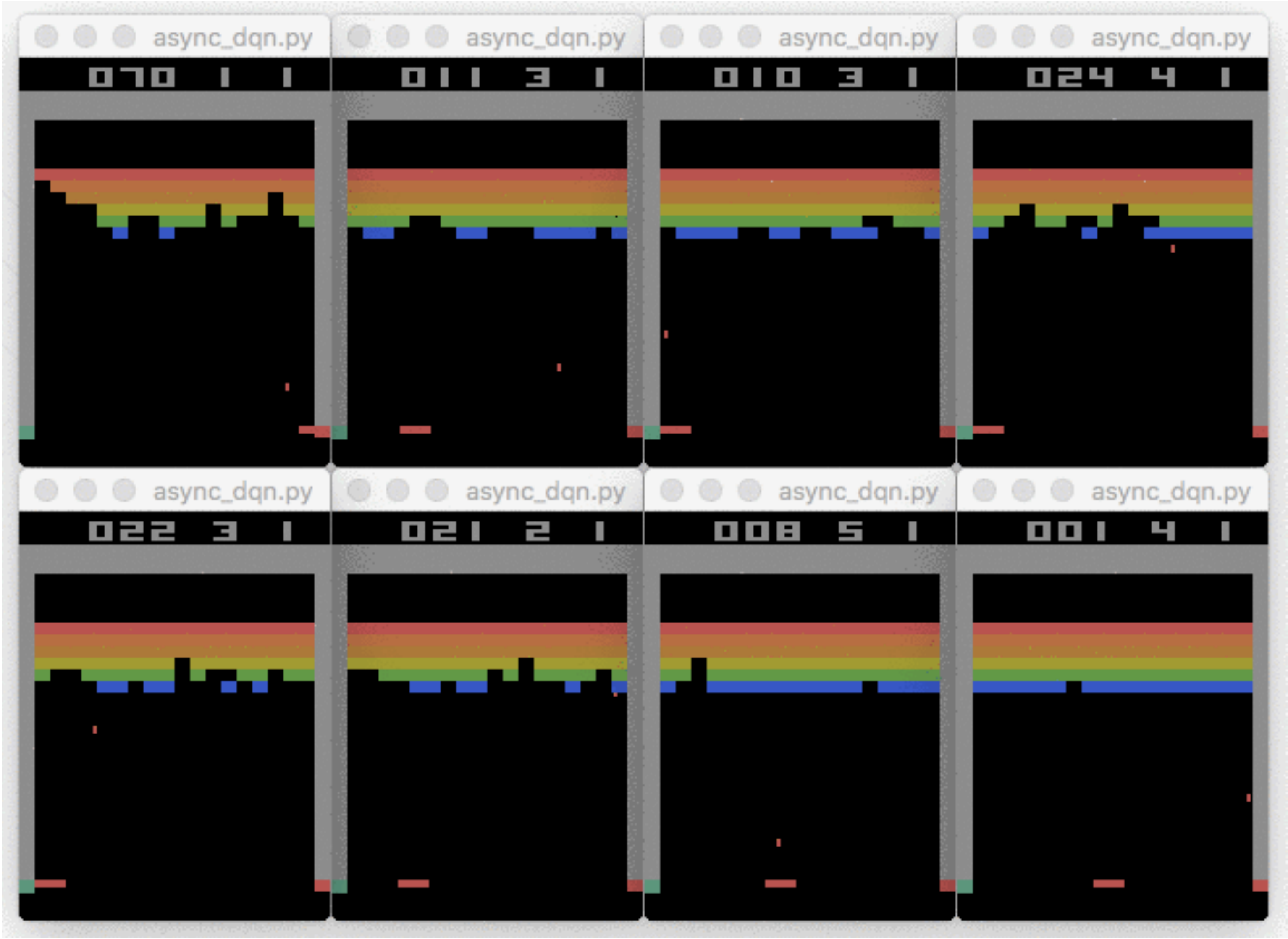
$$\mathcal{L}(w) = \mathbb{E}_{s,a,r,s' \sim \mathcal{D}} \left[\left(r + \gamma \max_{a'} Q(s', a', w^-) - Q(s, a, w) \right)^2 \right]$$

- Periodically update fixed parameters $w^- \leftarrow w$

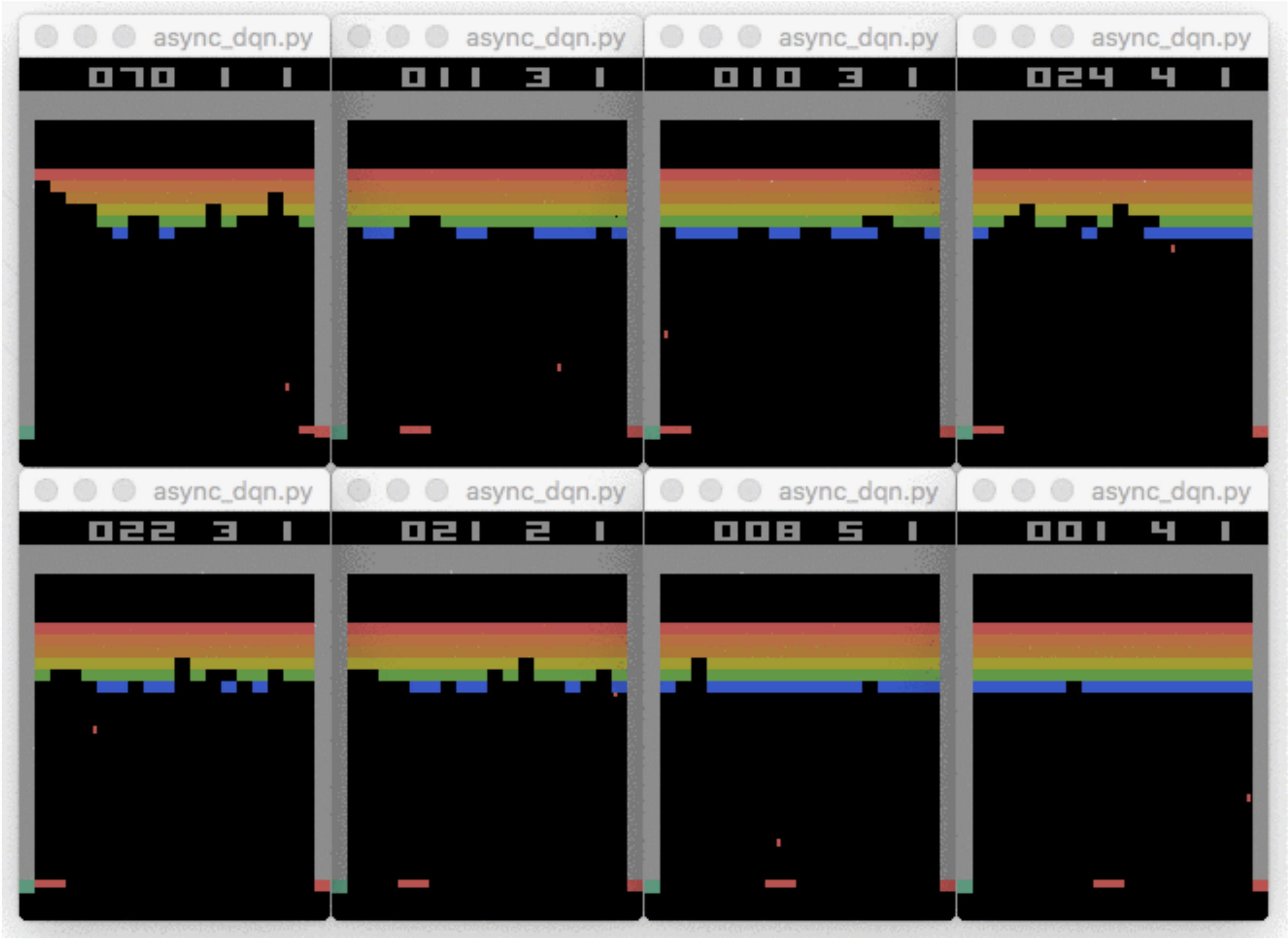
Clipped Rewards

- DQN clips the rewards to $[-1, +1]$
- This prevents Q-values from becoming too large
- Ensures gradients are well-conditioned
- Can't tell difference between small and large rewards

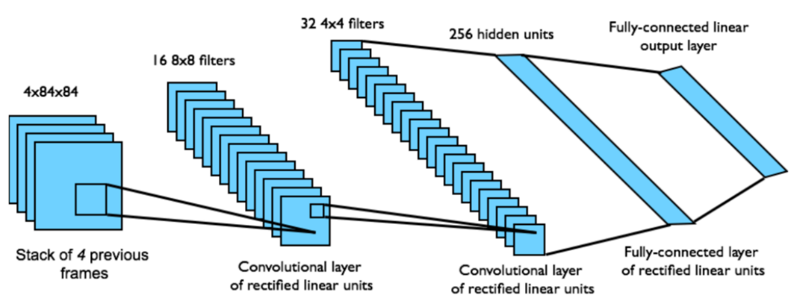
DQN for Atari Games



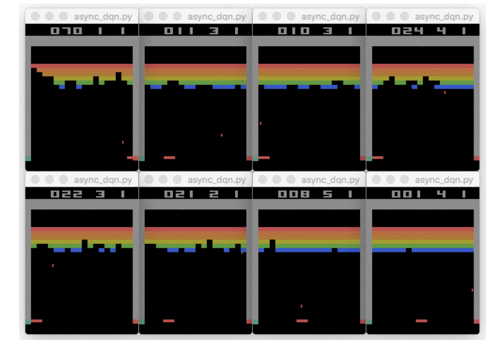
DQN for Atari Games



DQN for Atari Games



control



sampling
optimize

self-generated data

Replay Buffer

Published: 25 February 2015

Human-level control through deep reinforcement learning

Volodymyr Mnih, Koray Kavukcuoglu, David Silver, Andrei A. Rusu, Joel Veness, Marc G. Bellemare, Alex Graves, Martin Riedmiller, Andreas K. Fidjeland, Georg Ostrovski, Stig Petersen, Charles Beattie, Amir Sadik, Ioannis Antonoglou, Helen King, Dharshan Kumaran, Daan Wierstra, Shane Legg & Demis Hassabis

Nature **518**, 529–533 (2015) | Cite this article

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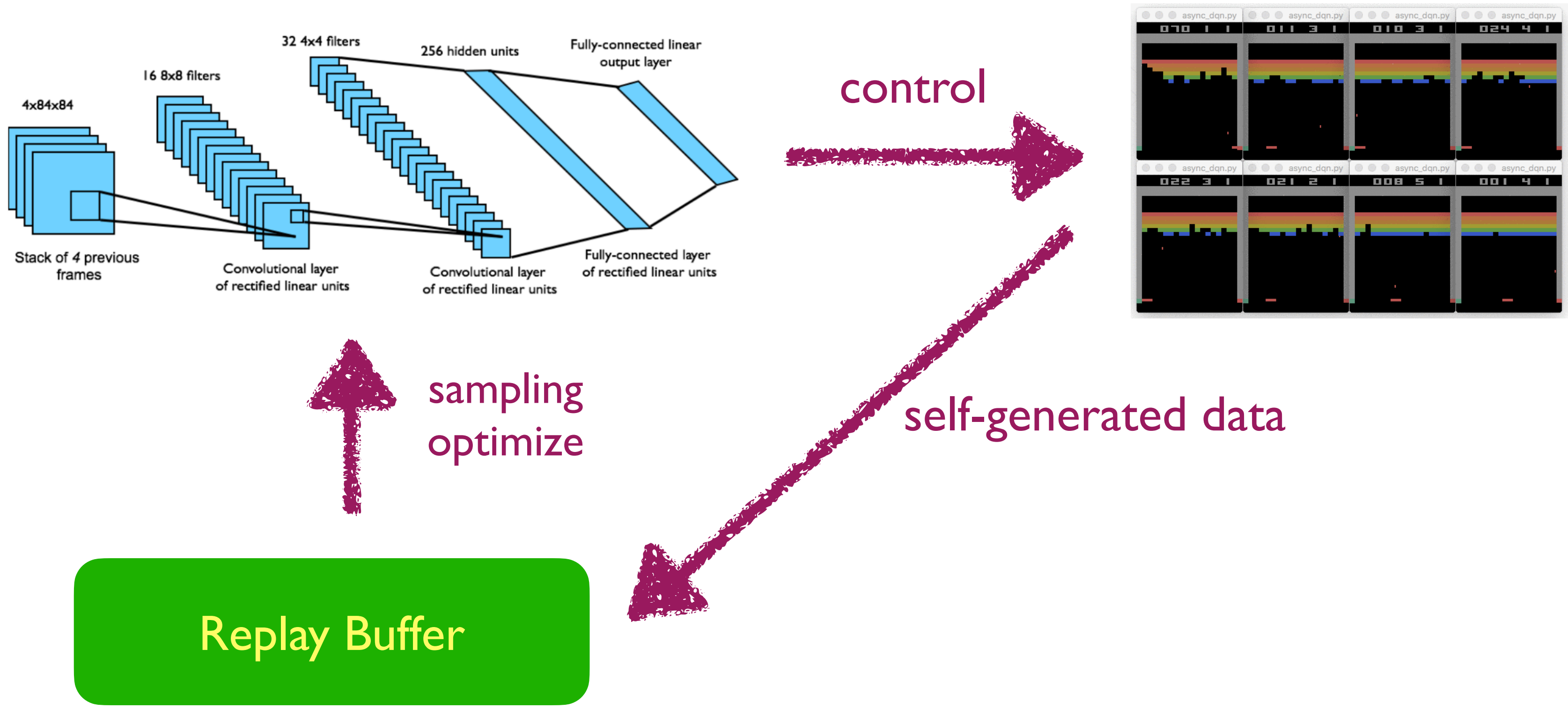
First break through of deep RL.

DQN for Atari Games



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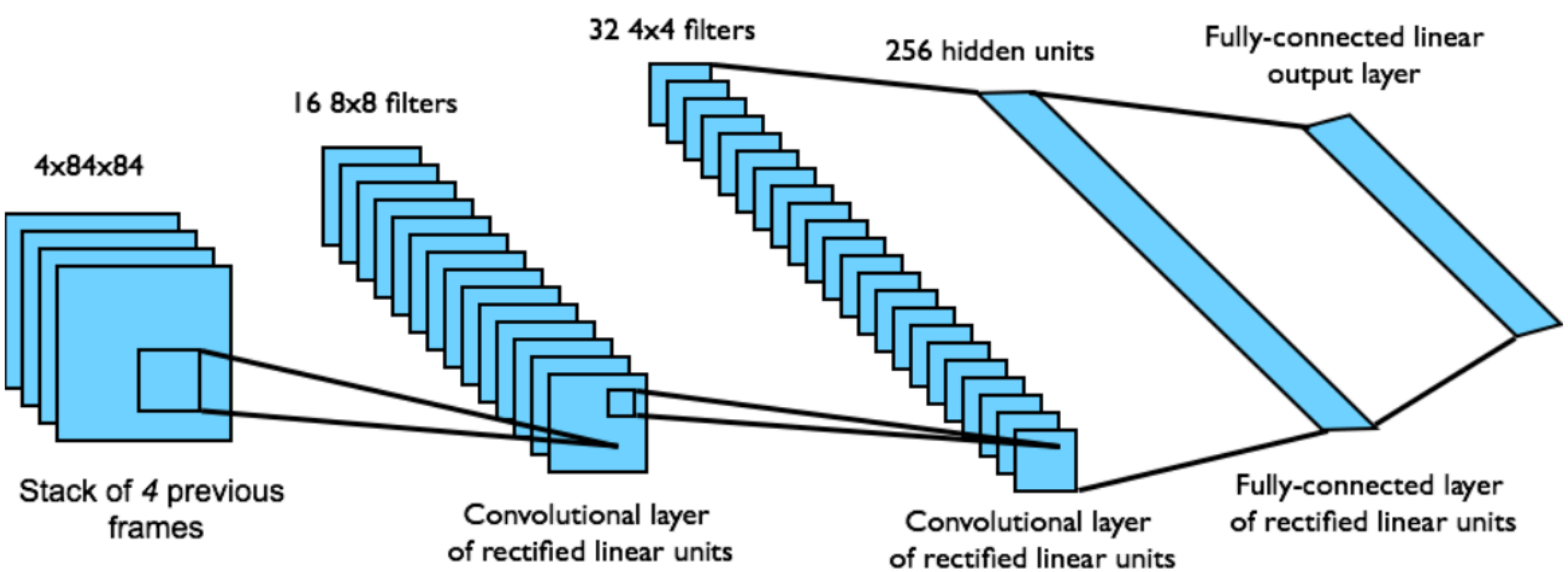
First break through of deep RL.

DQN for Atari Games

- End-to-end learning of values $Q(s, a)$ from pixels s
- Input state s is stack of raw pixels from last 4 frames
- Output is $Q(s, a)$ for 18 joystick/button positions
- Reward is change in score for that step

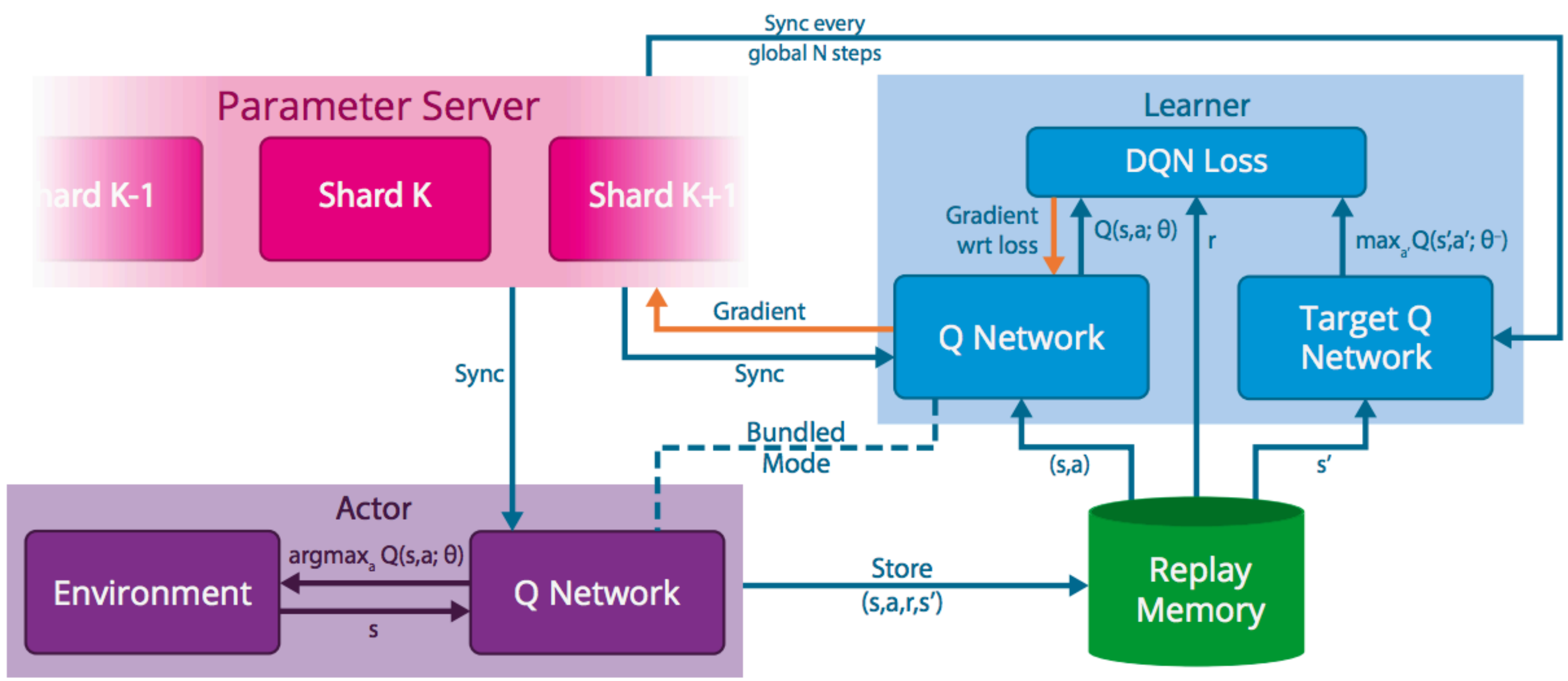


Network architecture and hyperparameters fixed across all games

[Mnih et al.]

Deep RL Architecture

Gorila (GOogle ReInforcement Learning Architecture)



- Parallel acting: generate new interactions
- Distributed replay memory: save interactions
- Parallel learning: compute gradients from replayed interactions
- Distributed neural network: update network from gradients

Slide courtesy: David Silver

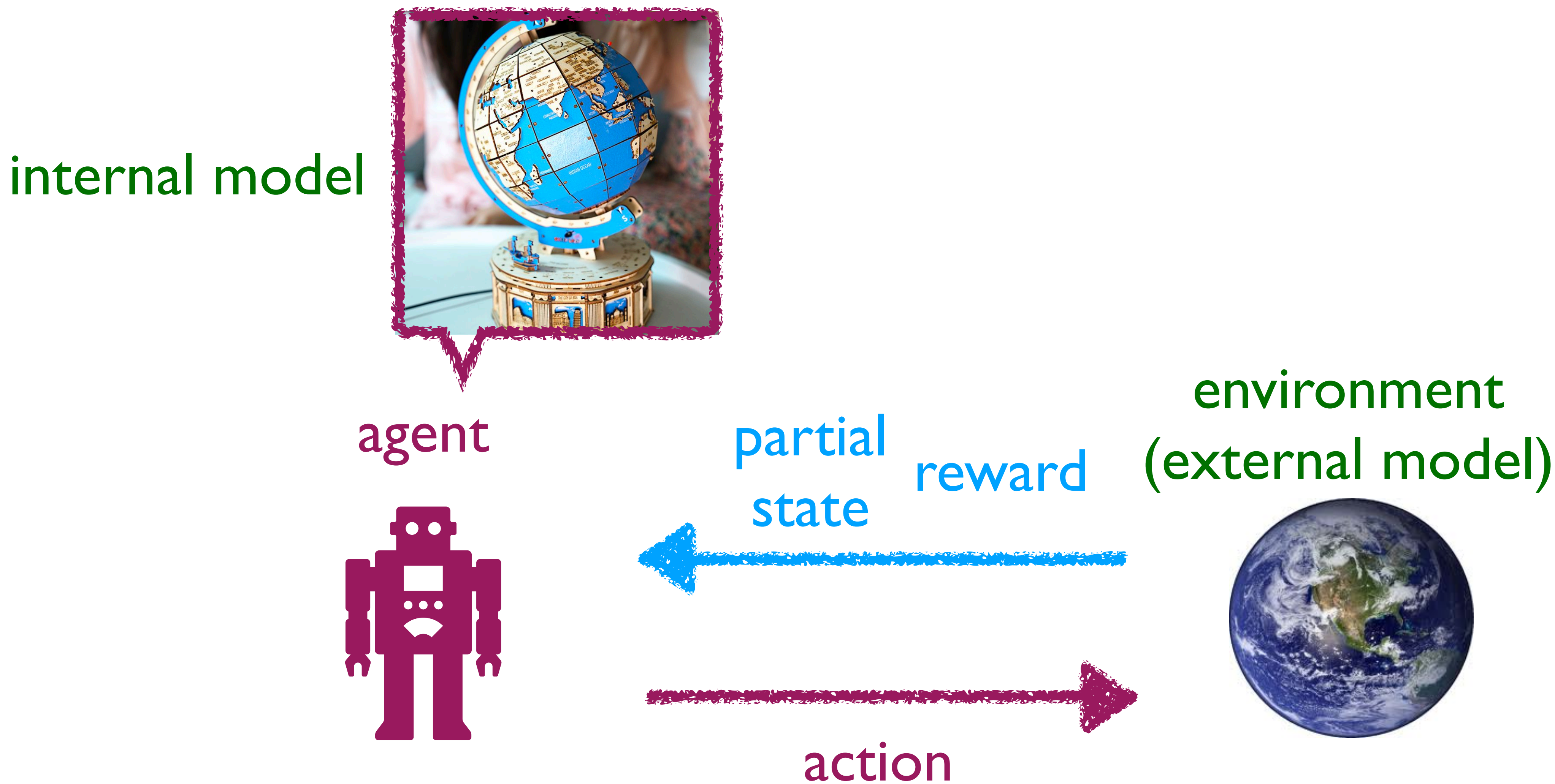
Reinforcement Learning

- Basics
  - Markov decision process
  - RL with known model
  - RL with unknown model
  - Policy gradient & actor-critic methods
- Deep reinforcement learning
- Integrating learning and planning
- RL from human preference
- Take-home messages

Slides link:

Internal and External Model

Models are crucial in decision making problems.
Whenever we have the external model or can obtain the internal model, we can combine the power of learning and planning (e.g. search, dynamic programming).



Integrating Learning and Planning

- Model-Free RL
  - No model
  - Learn value function (and/or policy) from real experience
- Model-Based RL (using Sample-Based Planning)
  - Learn a model from real experience
  - Plan value function (and/or policy) from simulated experience
- Dyna
  - Learn a model from real experience
  - Learn and plan value function (and/or policy) from real and simulated experience

Machine Learning Proceedings 1990

Proceedings of the Seventh International Conference, Austin, Texas, June 21–23, 1990

1990, Pages 216-224

Integrated Architectures for Learning, Planning, and Reacting Based on Approximating Dynamic Programming

Richard S. Sutton

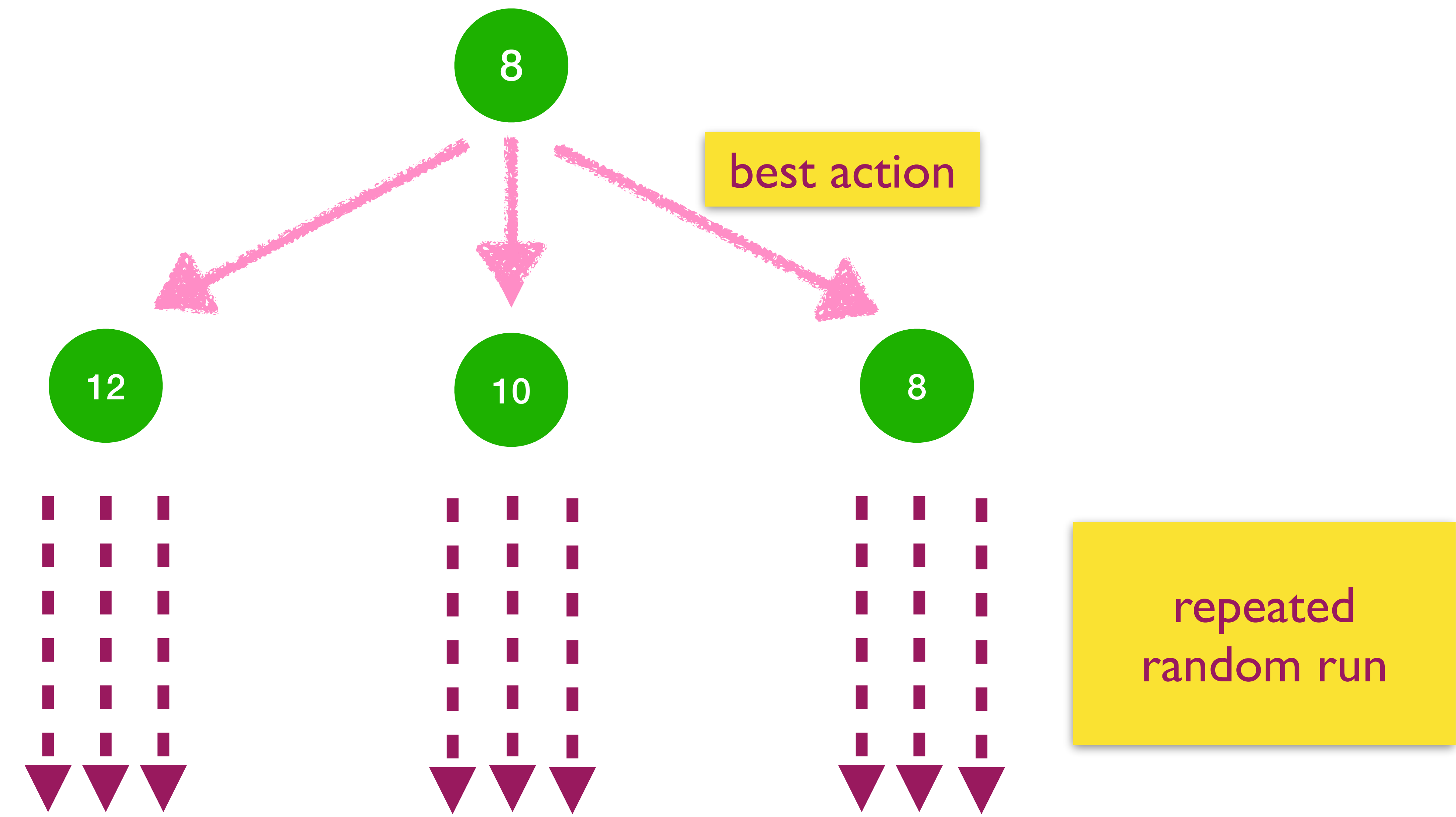
Planning by DP

Problem	Bellman Equation	Algorithm
Prediction	Bellman Expectation Equation	Iterative Policy Evaluation
Control	Bellman Expectation Equation + Greedy Policy Improvement	Policy Iteration
Control	Bellman Optimality Equation	Value Iteration

- Algorithms are based on state-value function $v_\pi(s)$ or $v_*(s)$
- Complexity $O(mn^2)$ per iteration, for m actions and n states
- Could also apply to action-value function $q_\pi(s, a)$ or $q_*(s, a)$
- Complexity $O(m^2n^2)$ per iteration

Slide courtesy: David Silver

Monte-Carlo Simulation

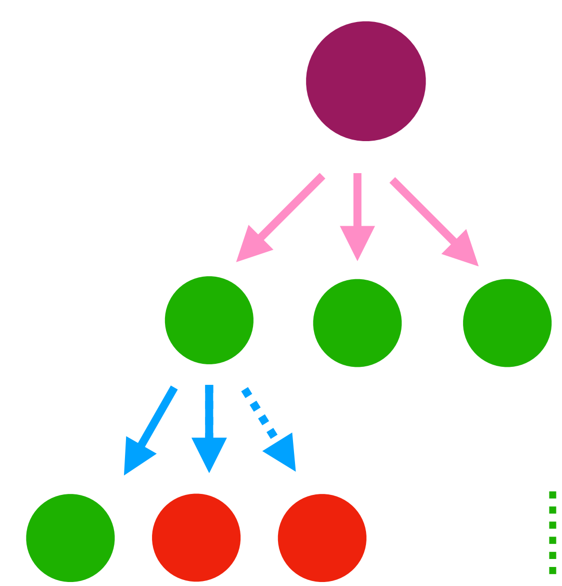


$\bar{U}_1 = 12$ $\bar{U}_2 = 10$ $\bar{U}_3 = 8$

Converge to true utility when the #run is sufficient!
But in real game playing, the time and space for simulation is limited.
We need a smart strategy to decide the order of simulation.

Monte-Carlo Tree Search (MCTS)

- Nodes of two kinds: visited node with unexpanded child, and unvisited node.



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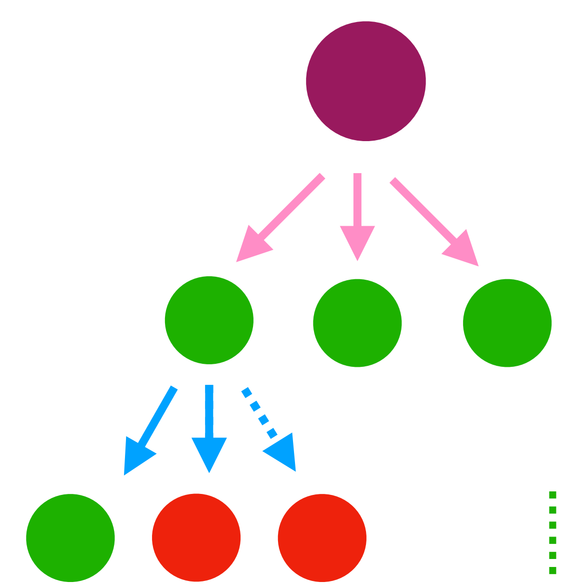
- During MCTS, we use two strategies: tree and default



Monte-Carlo Tree Search (MCTS)

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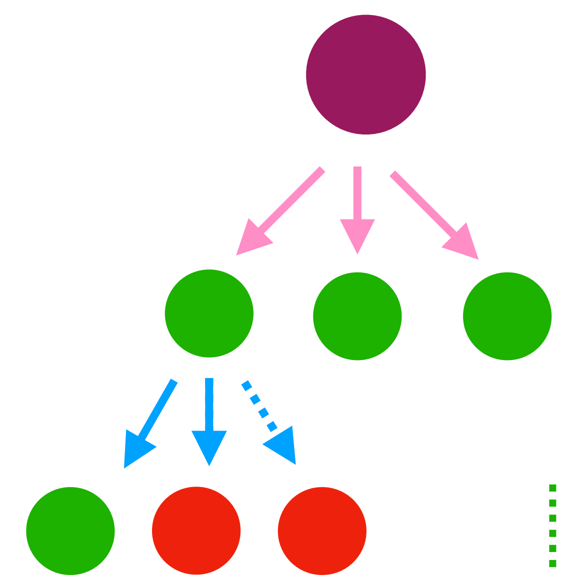
- Algorithm: repeat until time or space limit:



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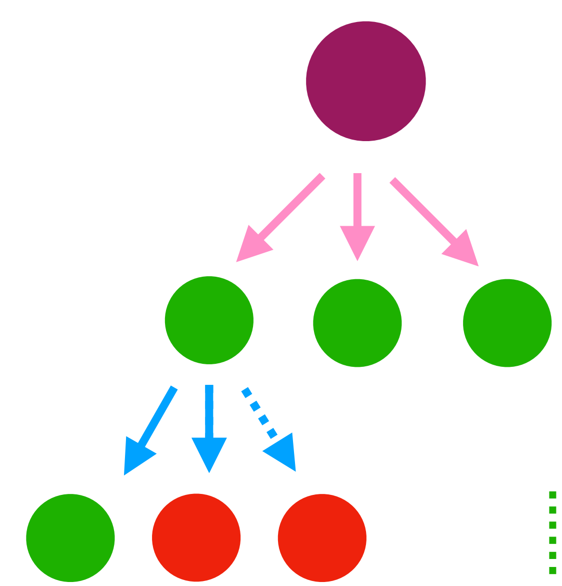

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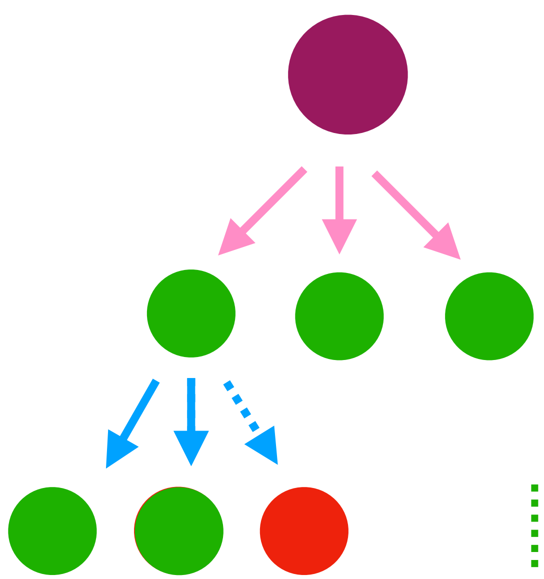

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- During MCTS, we use two strategies: tree and default
- Algorithm: repeat until time or space limit:
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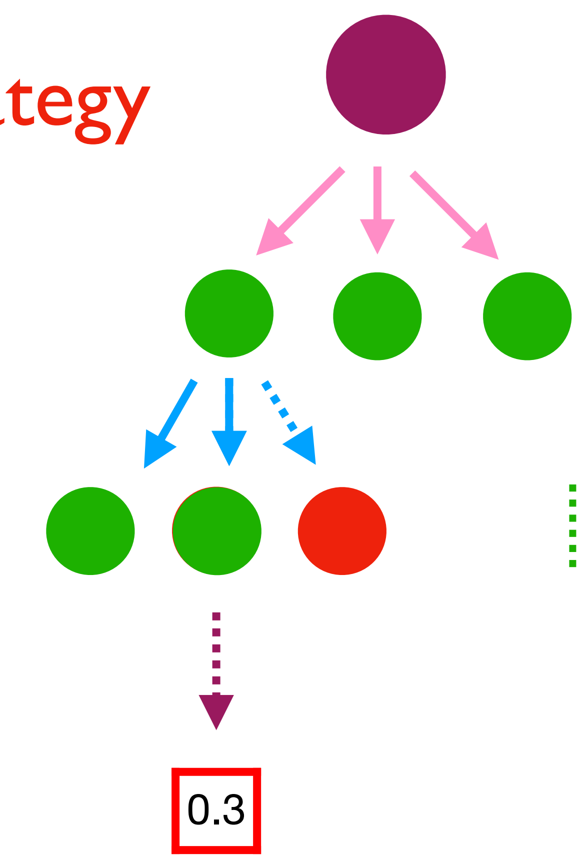
  - Expansion: expand an unvisited child and put into
  - Simulation: simulate down using default strategy



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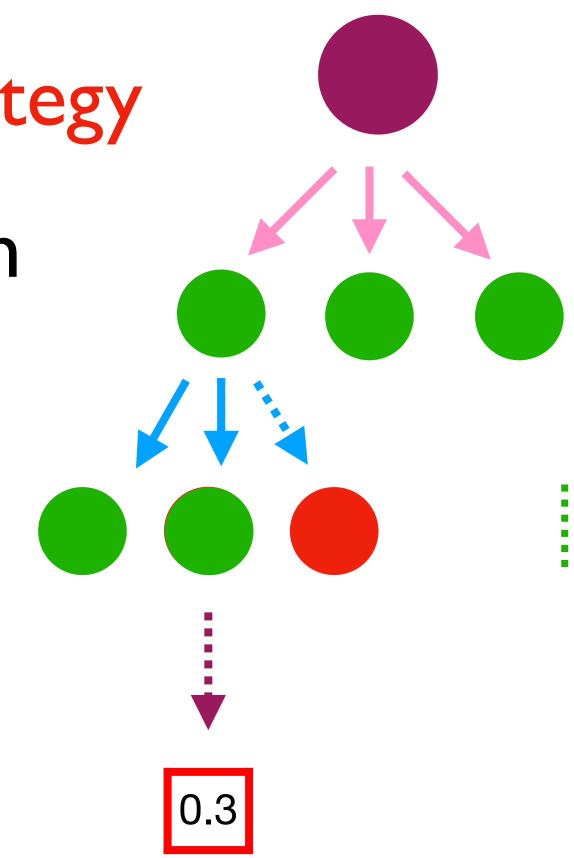
- Algorithm: repeat until time or space limit:
  - Selection: choose one node among using tree strategy
  - Expansion: expand an unvisited child and put into
  - Simulation: simulate down using default strategy
  - Update: update MC estimation through path



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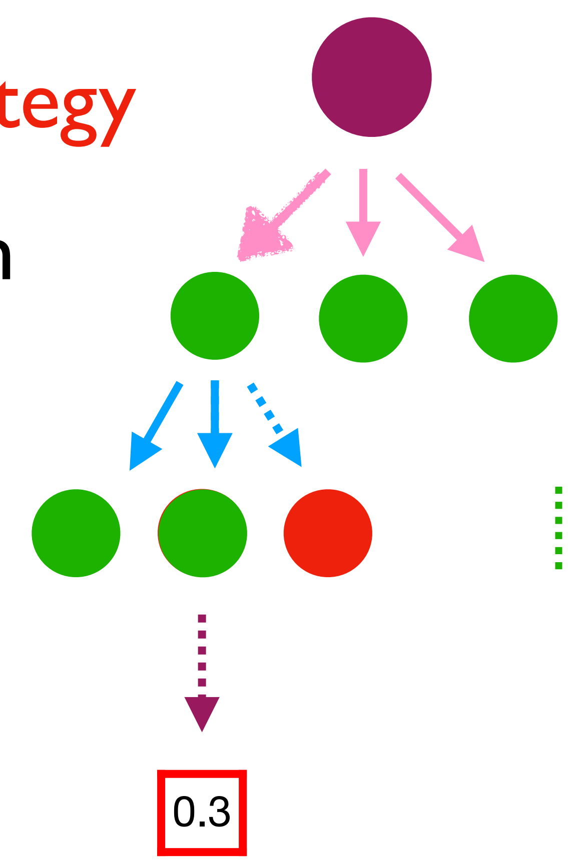
- Output the best action to play



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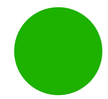
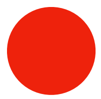
- Nodes of two kinds:  visited node with unexpanded child, and  unvisited node.
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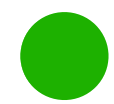
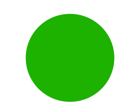
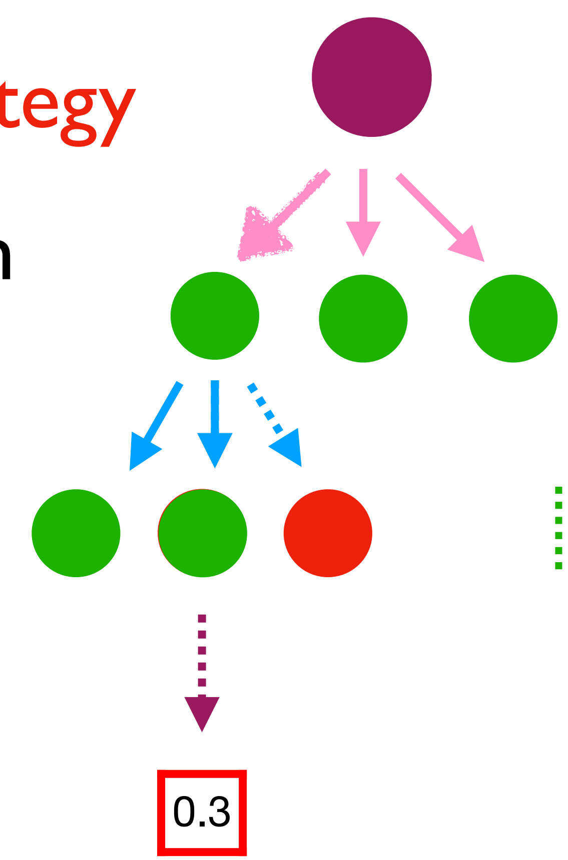
  - Selection: choose one node among  using tree strategy
  - Expansion: expand an unvisited child and put into 
  - Simulation: simulate down using default strategy
  - Update: update MC estimation through path
- Output the best action to play

The default strategy is usually random play
The tree strategy is essential:
Deciding the order of search



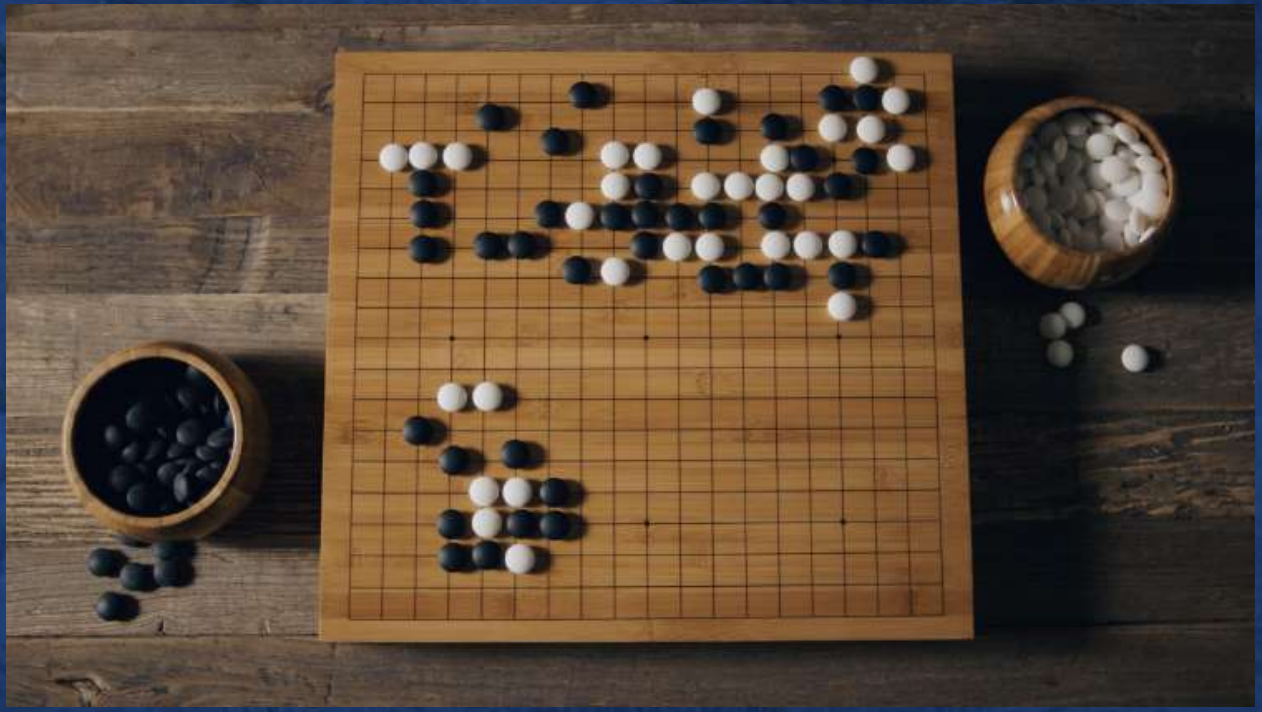
AlphaGo:
Integration of Learning & Planning

Why is Go hard for computers to play?

Game tree complexity = b^d

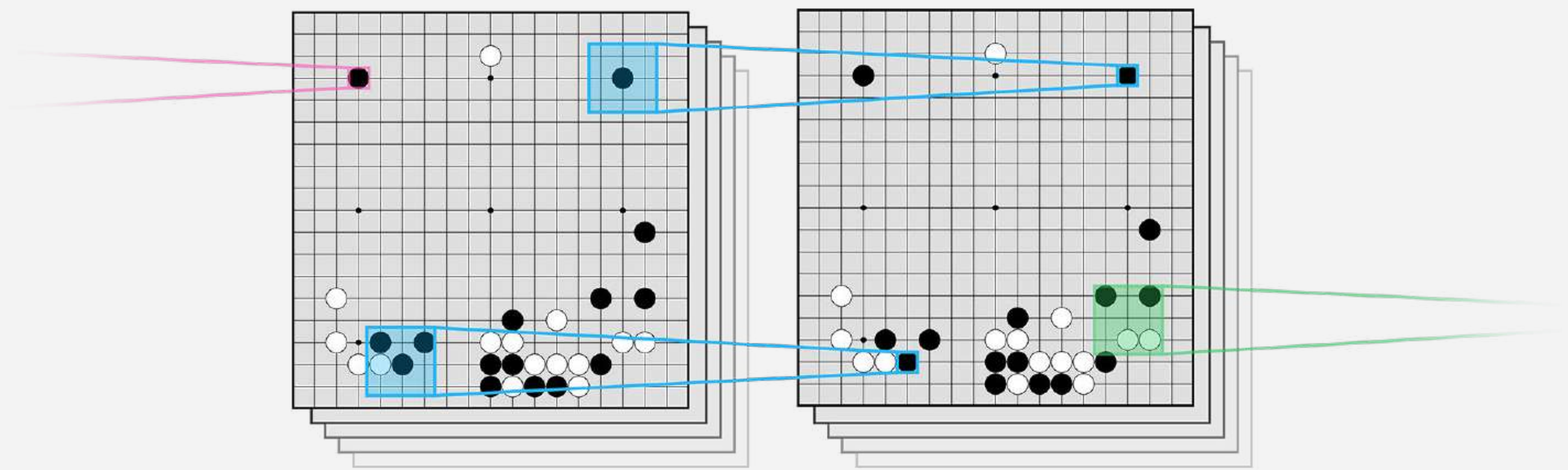
Brute force search intractable:

1. Search space is huge
2. “Impossible” for computers to evaluate who is winning



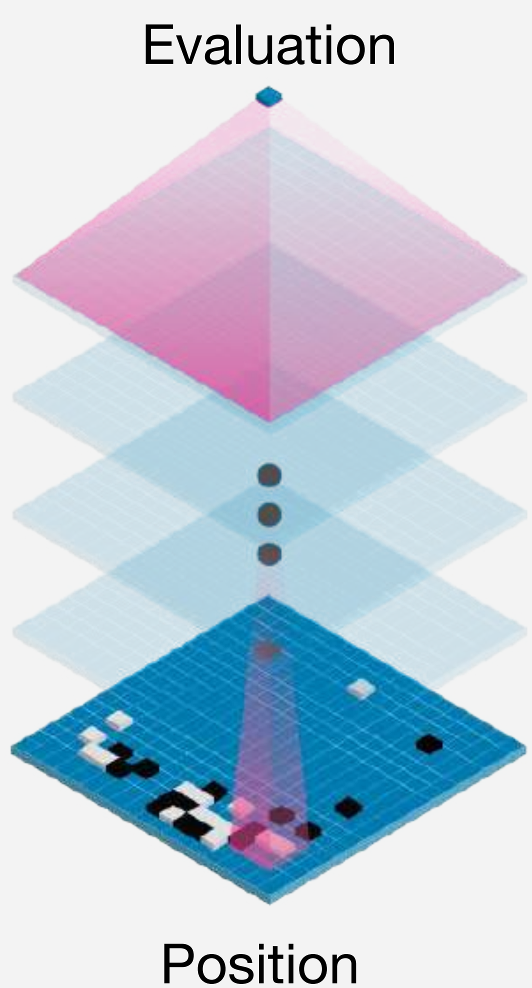
Google DeepMind

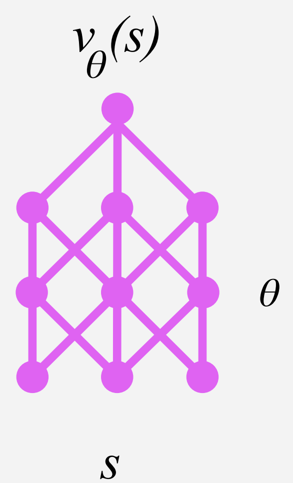
Convolutional neural network



AlphaGo introduces three conv. nets for learning.
Use images of board as state inputs.

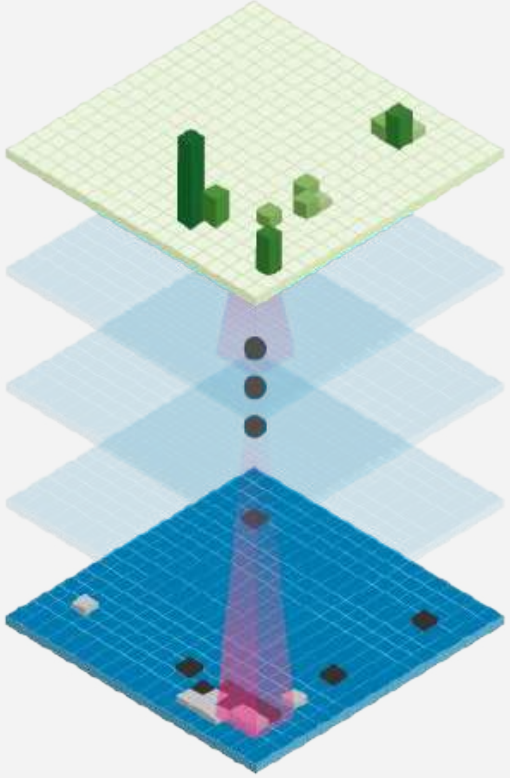
Value network

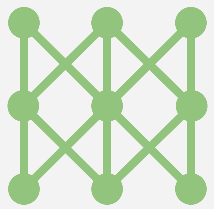




The value function of RL.

Policy network

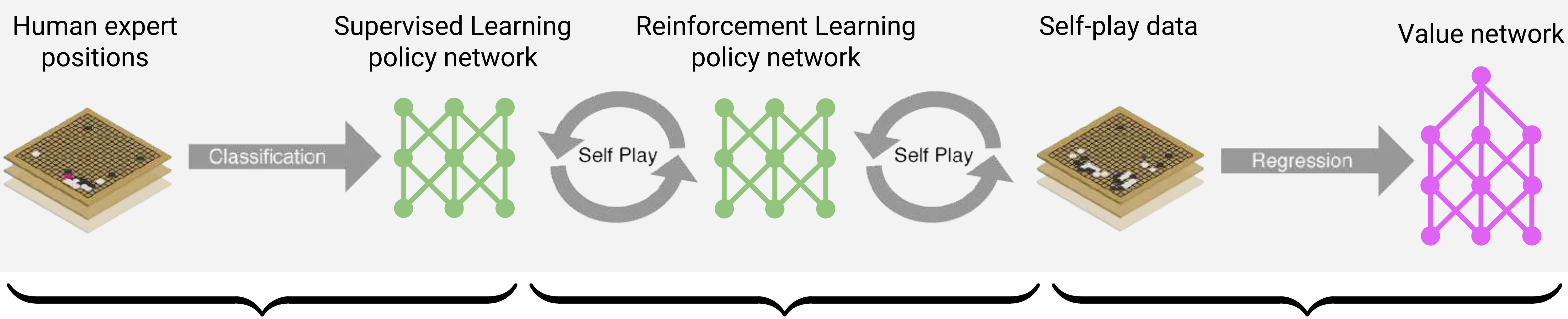




Google DeepMind

Policy network of RL.

Neural network training pipeline



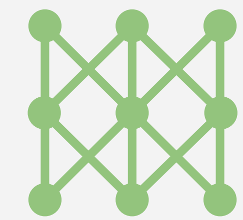
First stage:
Supervised classification

Second stage:
Policy gradient RL

Second stage:
Supervised regression

Learning: First Stage

Supervised learning of policy networks



Policy network: 12 layer convolutional neural network

Training data: 30M positions from human expert games (KGS 5+ dan)

Training algorithm: maximise likelihood by stochastic gradient descent

$$\Delta\sigma \propto \frac{\partial \log p_\sigma(a|s)}{\partial \sigma}$$

Training time: 4 weeks on 50 GPUs using Google Cloud

Results: 57% accuracy on held out test data (state-of-the art was 44%)

Google DeepMind

Learning: Second Stage

Reinforcement learning of policy networks

Policy network: 12 layer convolutional neural network

Training data: games of self-play between policy network

Training algorithm: maximise wins z by policy gradient reinforcement learning

$$\Delta\sigma \propto \frac{\partial \log p_\sigma(a|s)}{\partial \sigma} z$$

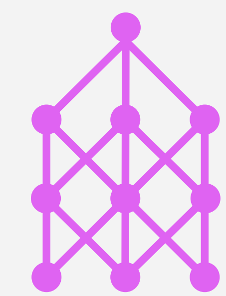
Training time: 1 week on 50 GPUs using Google Cloud

Results: 80% vs supervised learning. Raw network ~3 amateur dan.

Google DeepMind

Learning: Third Stage

Reinforcement learning of value networks



Value network: 12 layer convolutional neural network

Training data: 30 million games of self-play

Training algorithm: minimise MSE by stochastic gradient descent

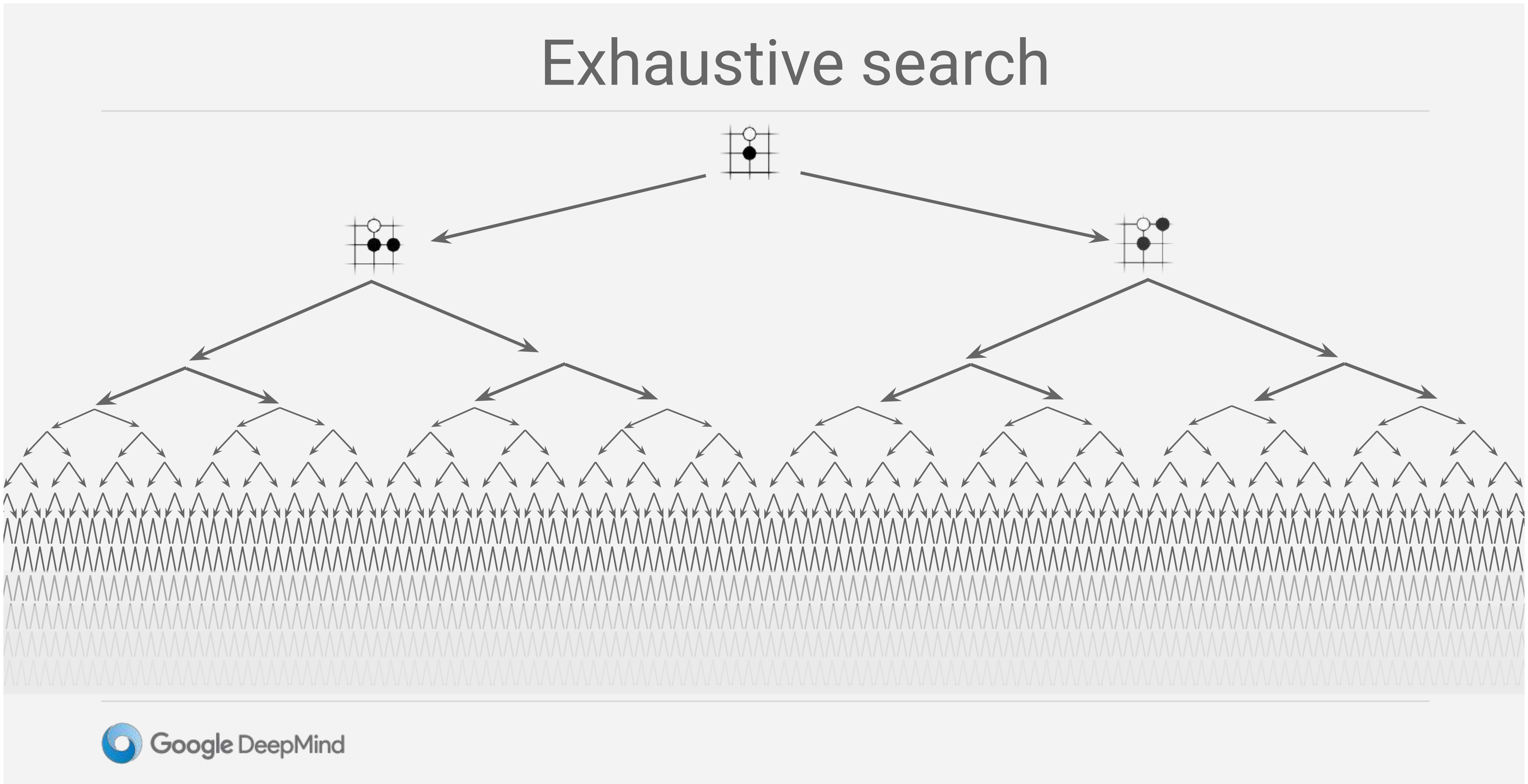
$$\Delta\theta \propto \frac{\partial v_\theta(s)}{\partial\theta}(z - v_\theta(s))$$

Training time: 1 week on 50 GPUs using Google Cloud

Results: First strong position evaluation function - previously thought impossible

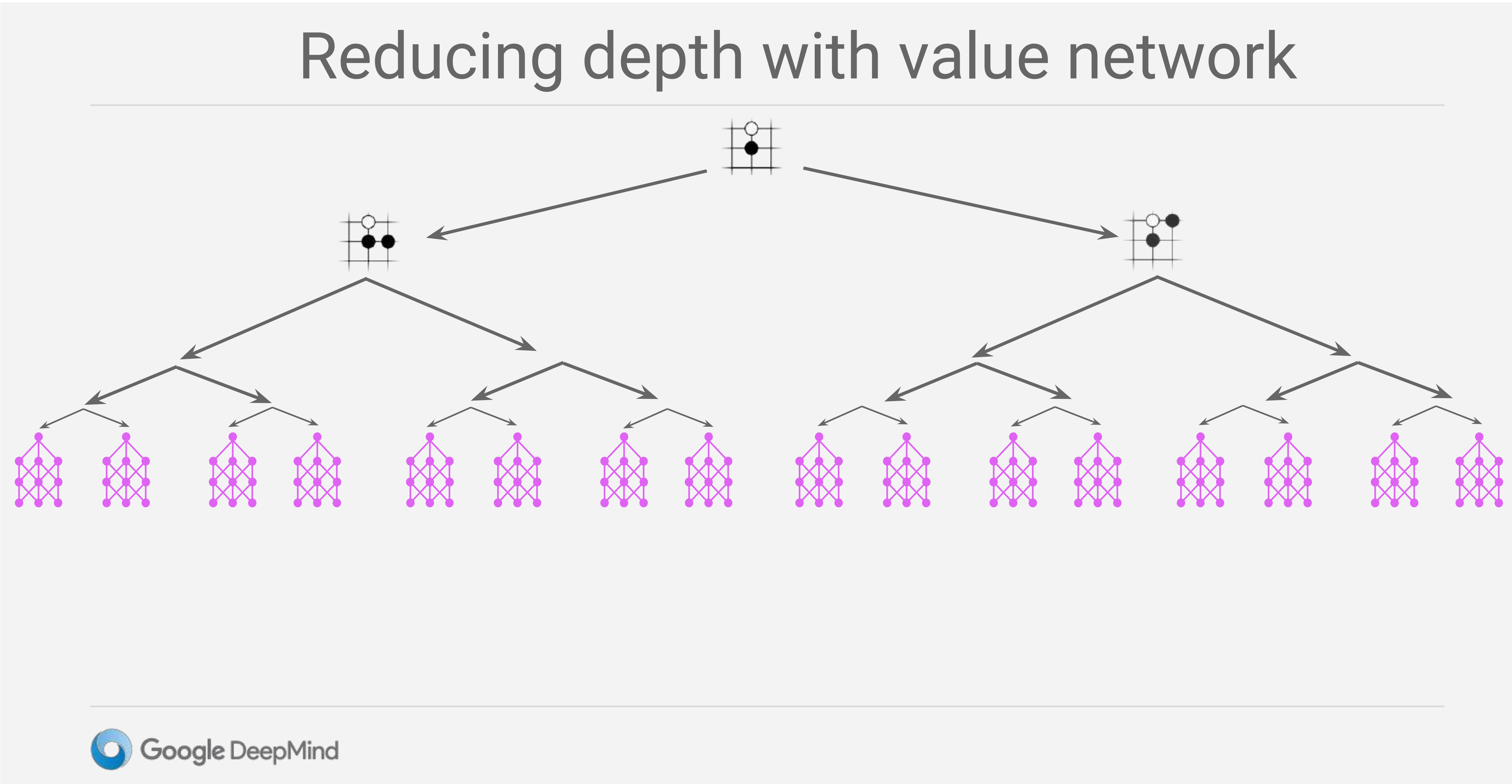
Google DeepMind

Real Play: MCTS



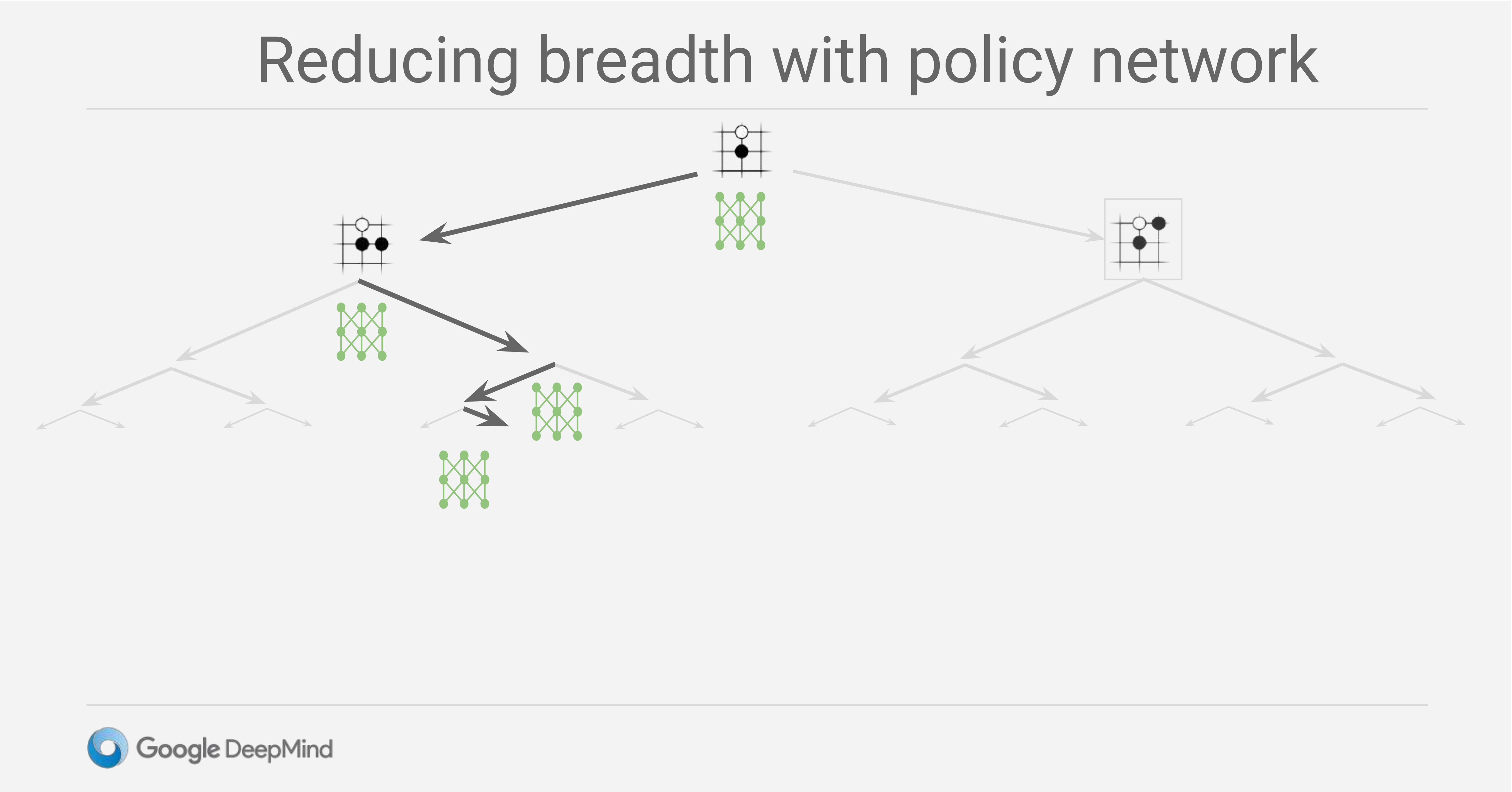
Two key steps:
node expansion and repeated random simulation

Real Play: MCTS



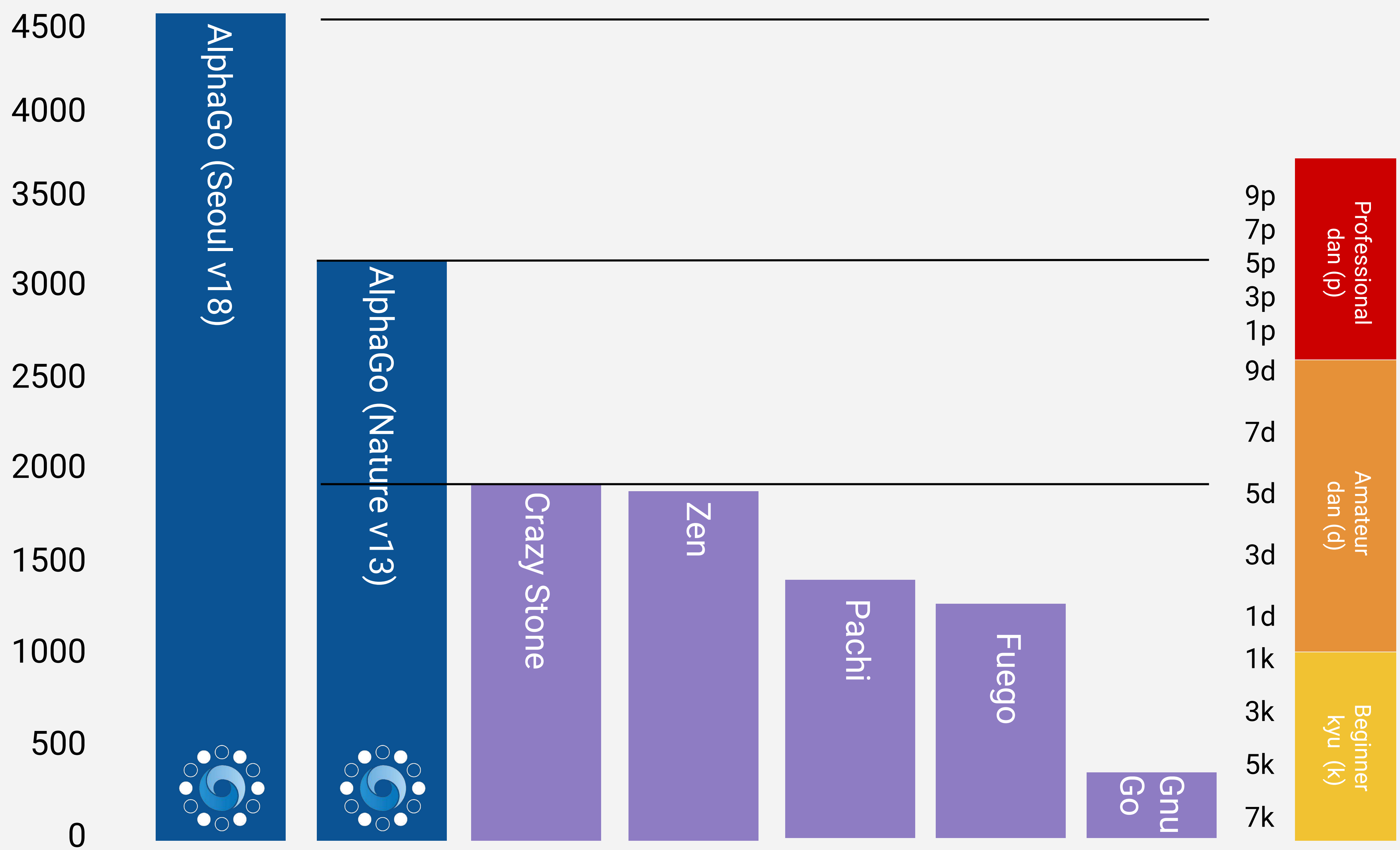
With value network, we can expand fewer depth
since the value of nodes can also be obtained from the value network.

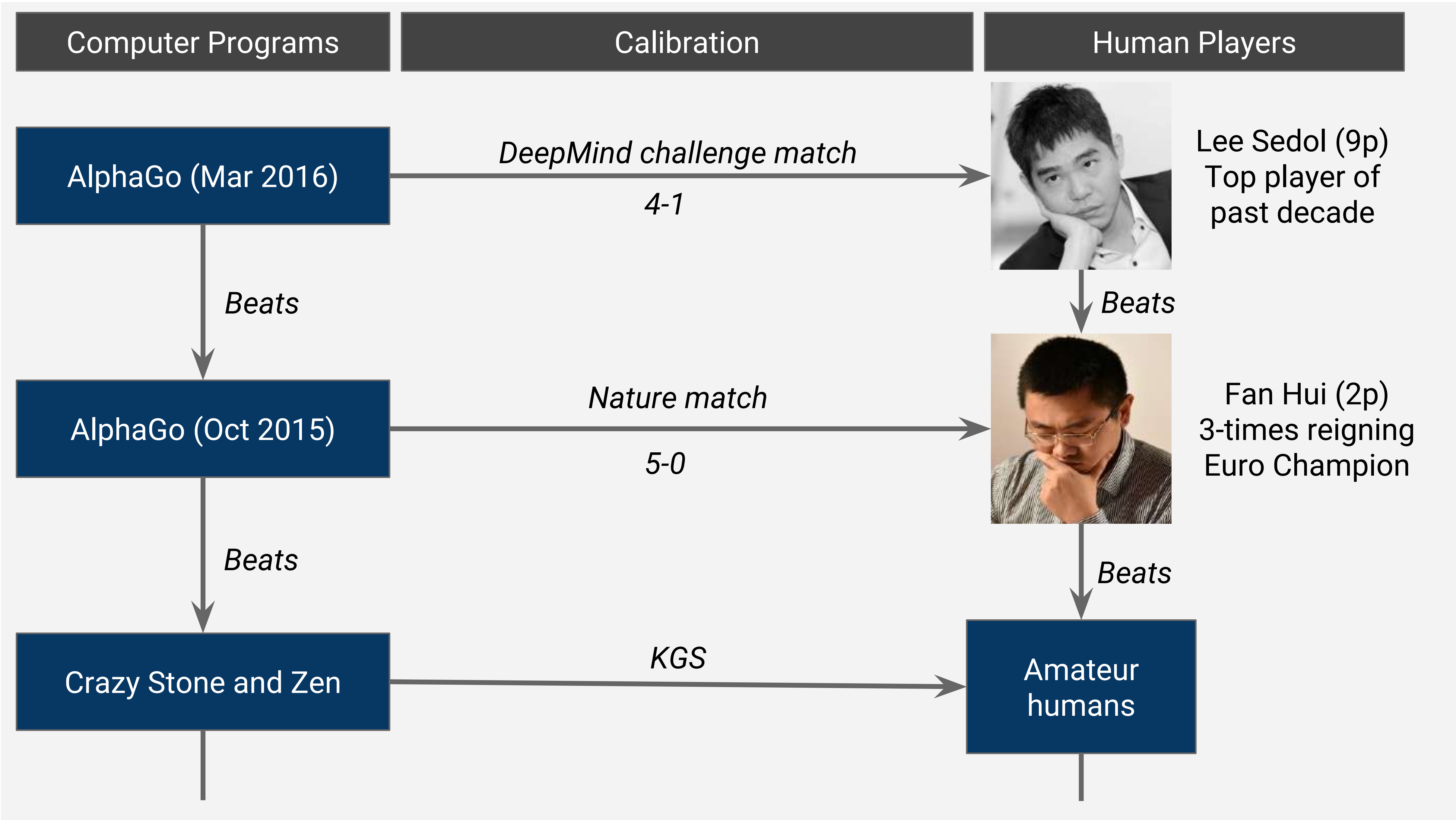
Real Play: MCTS



With policy network, we can simulate with fewer times
since the simulation can be guided by policy network over random play.

Evaluating AlphaGo against computers




Computer Programs
Calibration
Human Players
AlphaGo (Mar 2016)
DeepMind challenge match
4-1
Lee Sedol (9p)
Top player of
past decade
Beats
Beats
AlphaGo (Oct 2015)
Nature match
5-0
Fan Hui (2p)
3-times reigning
Euro Champion
Beats
Beats
Crazy Stone and Zen
KGS
Amateur
humans

Reinforcement Learning

- Basics
  - Markov decision process
  - RL with known model
  - RL with unknown model
  - Policy gradient & actor-critic methods
- Deep reinforcement learning
- Integrating learning and planning
- RL from human preference
- Take-home messages

Slides link:

Aligning Language Models to Follow Instructions

Prompt *Explain the moon landing to a 6 year old in a few sentences.*

Completion GPT-3

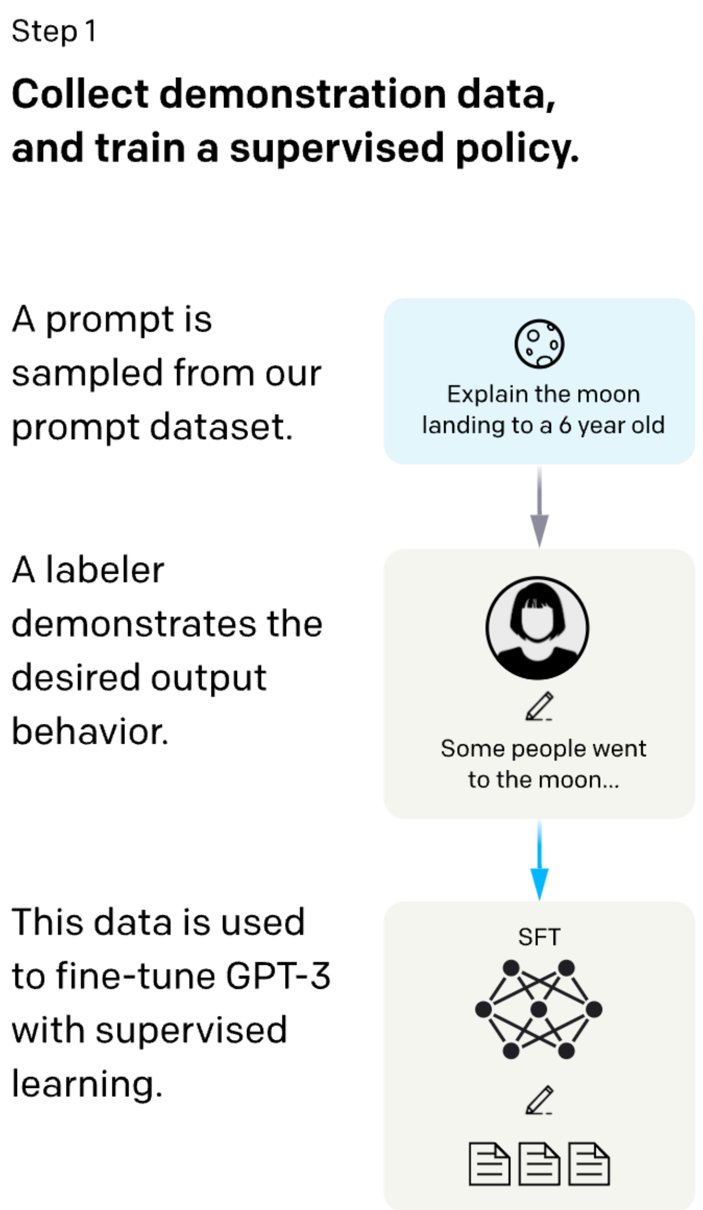
```
Explain the theory of gravity to a 6 year old.

Explain the theory of relativity to a 6 year old in a few sentences.

Explain the big bang theory to a 6 year old.

Explain evolution to a 6 year old.
```

InstructGPT

```
People went to the moon, and they took pictures of what they saw, and sent
them back to the earth so we could all see them.
```

https://openai.com/research/instruction-following#sample1

Fine-Tune GPT by RL



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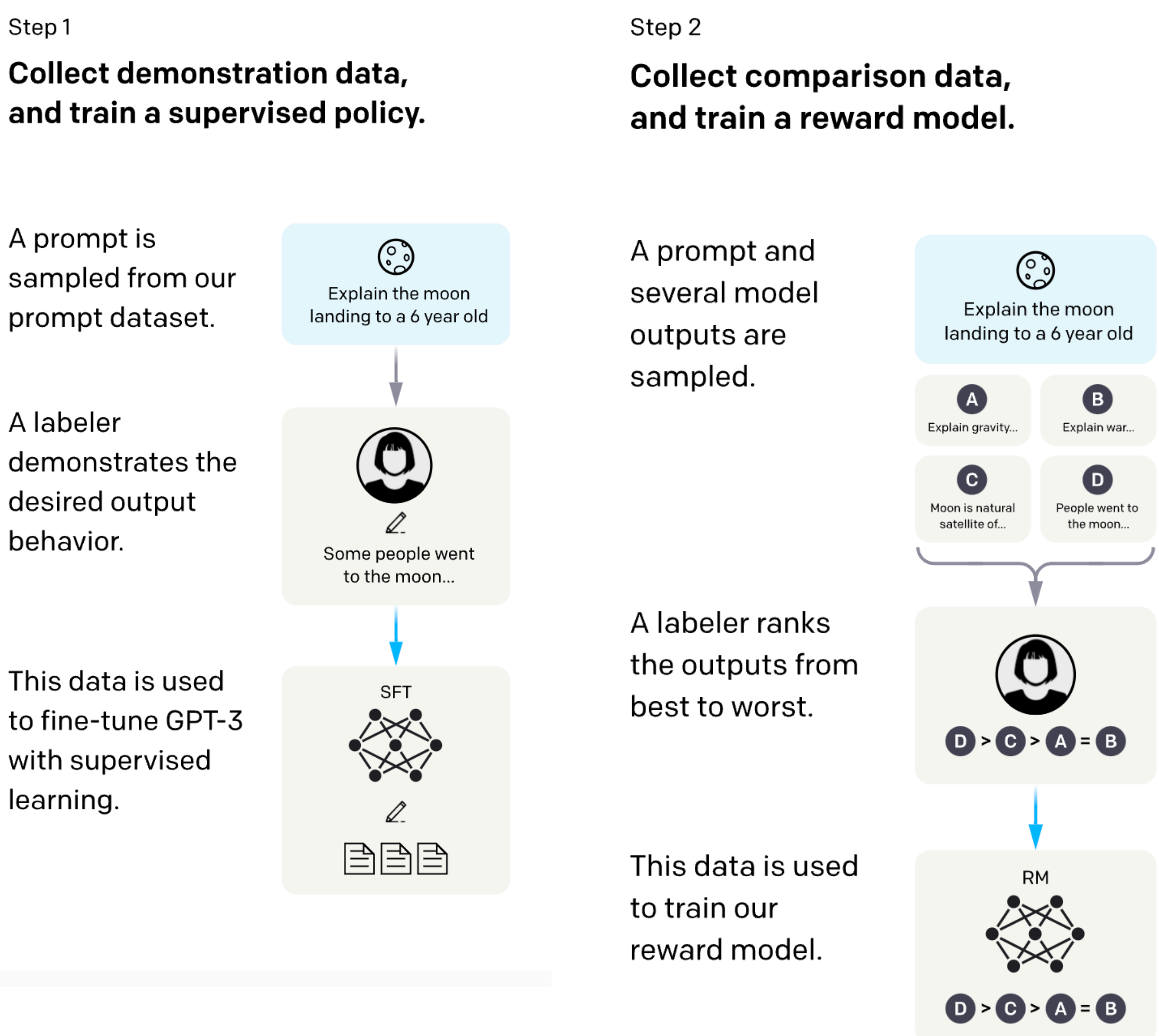

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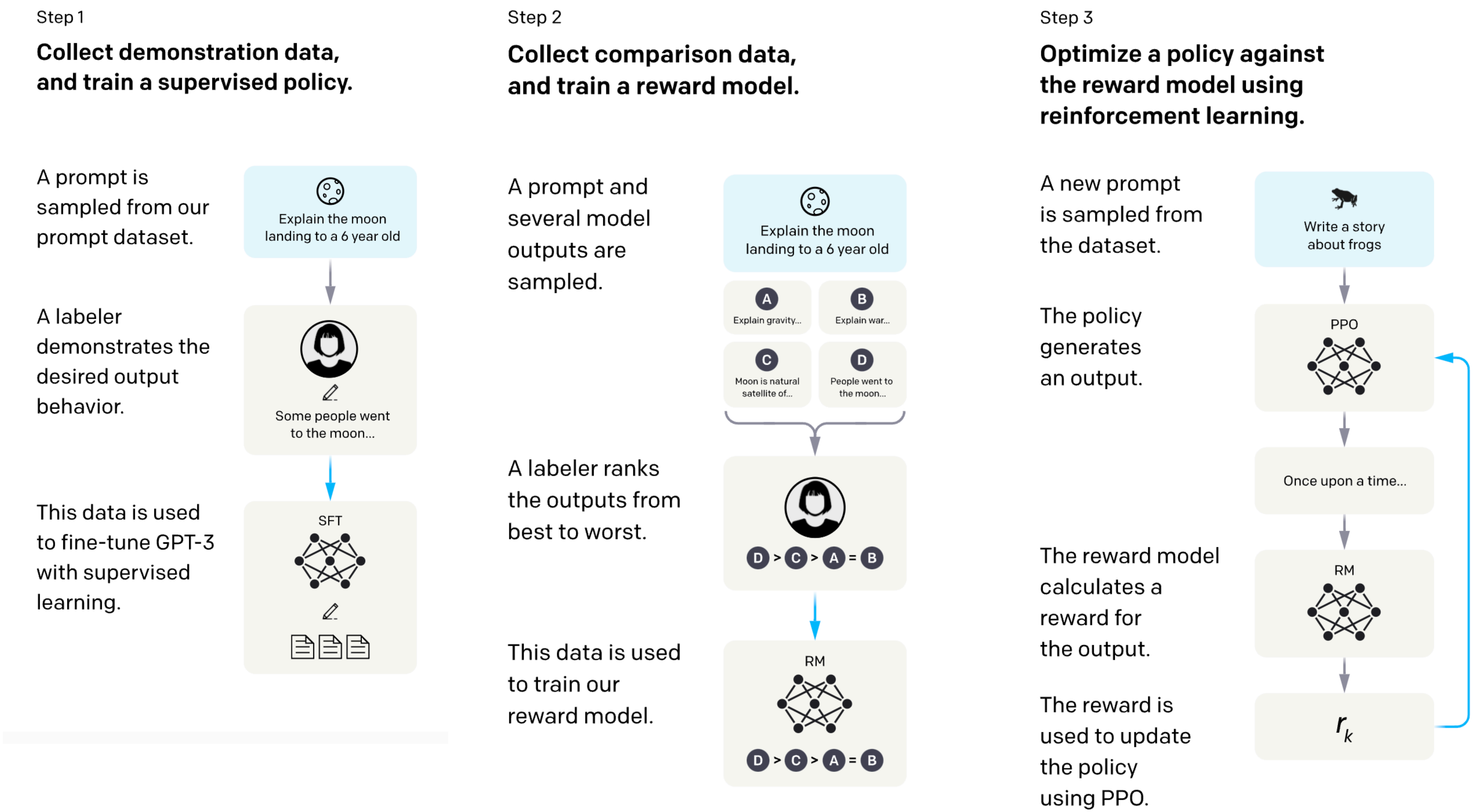

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Learning Reward Function from Human Preference

- Reward function for evaluating behavior segments: $r(o, a)$

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- Turn reward function into classifier to estimate the preference:

$$\hat{P}\left[\sigma^1 \succ \sigma^2\right] = \frac{\exp \sum \hat{r}\left(o_t^1, a_t^1\right)}{\exp \sum \hat{r}(o_t^1, a_t^1) + \exp \sum \hat{r}(o_t^2, a_t^2)}.$$

The Bradley–Terry model [1952]

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Why not let human directly label single segment?

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Take-Home Messages

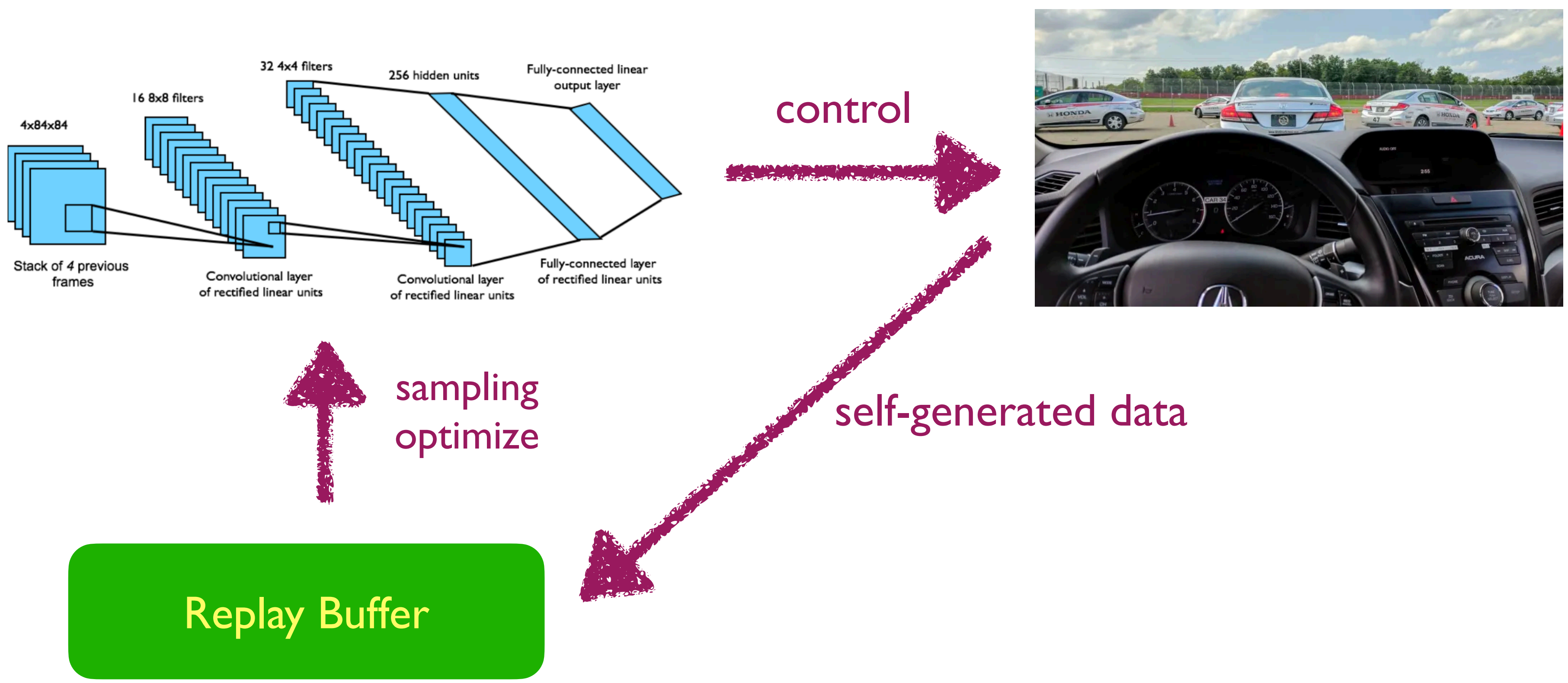
- Reinforcement learning solves decision-making problems by interaction with the environment.
- Markov decision process models the decision problem, when full information is known, dynamical programming can be used.
- Value function based RL utilizes MC or TD estimate of the value function.

Take-Home Messages

- To solve large-scale RL problems, functional approximation of value functions or policies is essential.
- Policy gradient & actor-critic: direct learning of policies. Usually more efficient for deep RL.
- Large-scale RL systems: Integrating learning and planning & RL from human preference.
- Next-steps of RL?

Why hasn't DRL solved many real-world problems?

- Deep RL usually does not use (learn) a world model.



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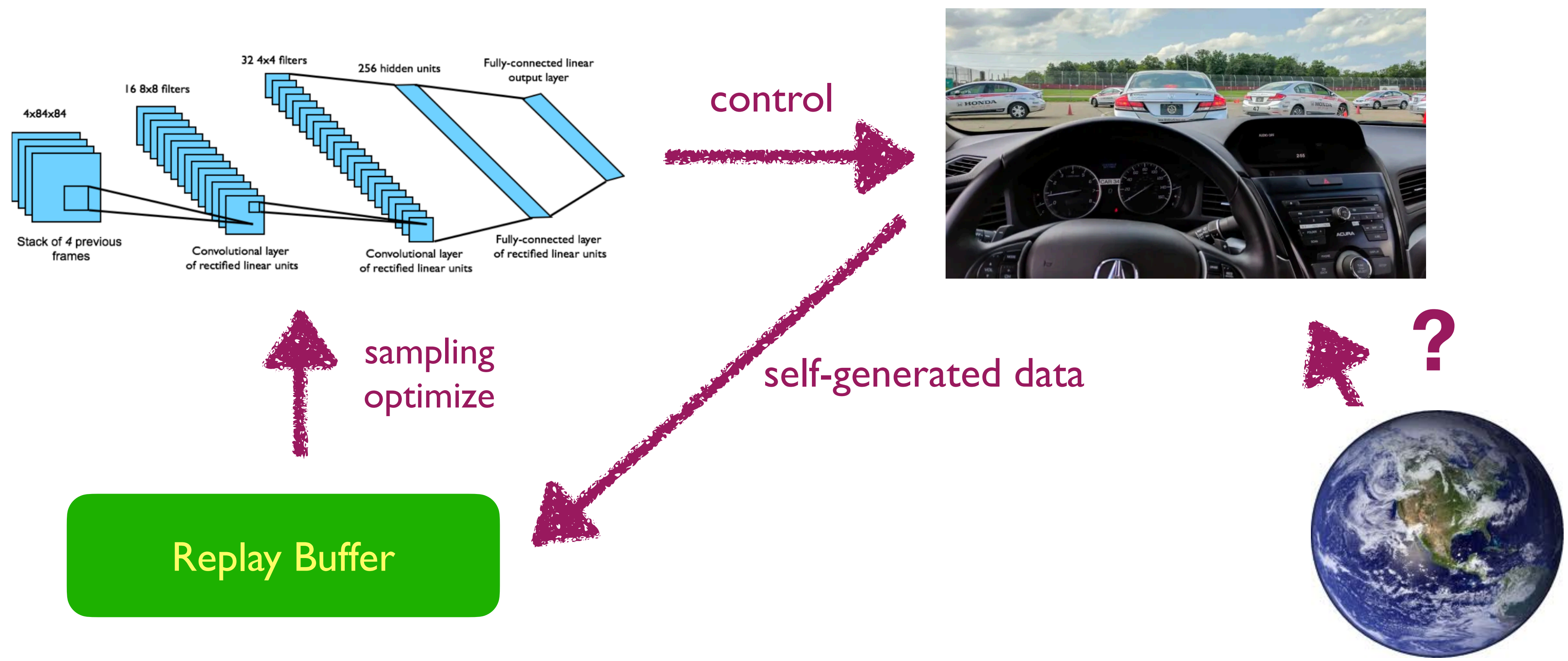

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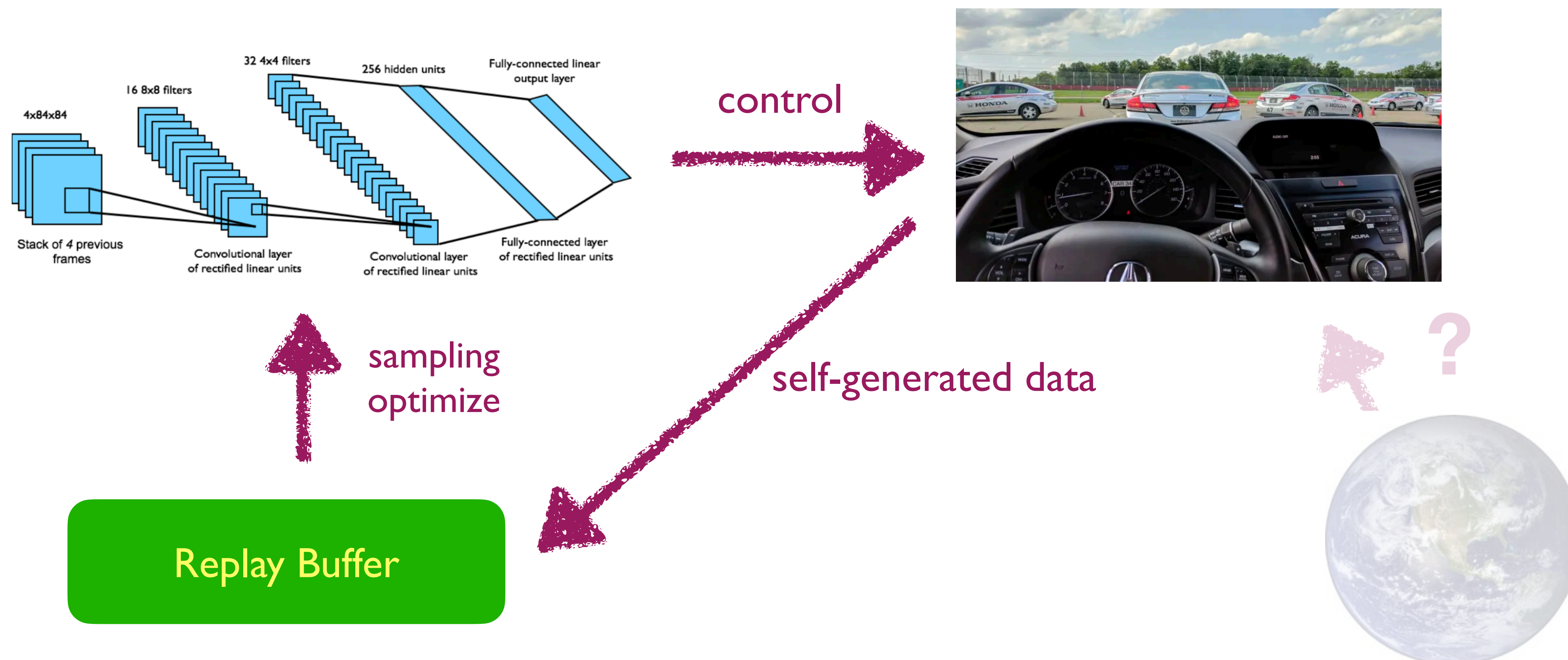

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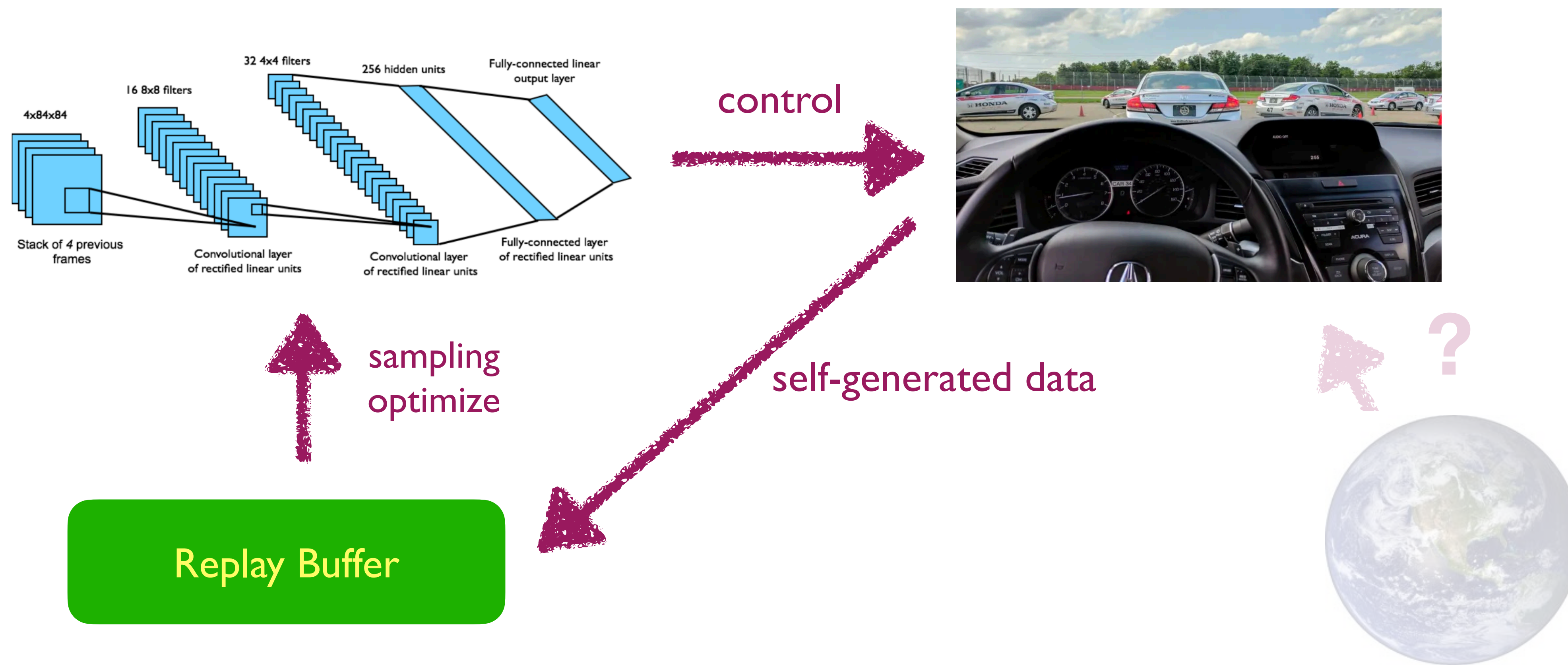

Without a world model, learning from self-generated data requires many trial-and-errors in the real world.
Large cost. Bad generalization to new task.

Model-Based RL

Scenarios of decision making:

Model-Based RL

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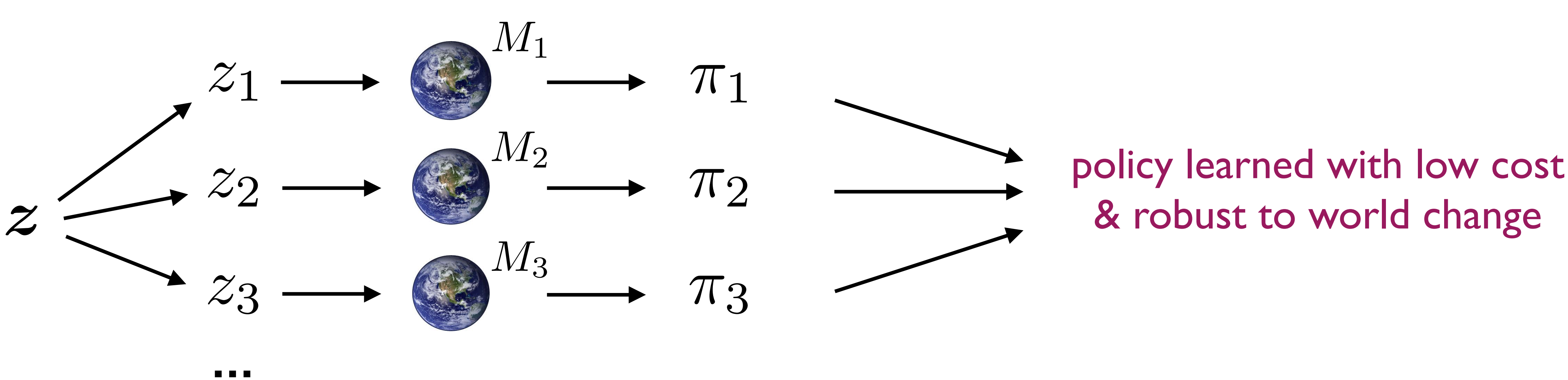

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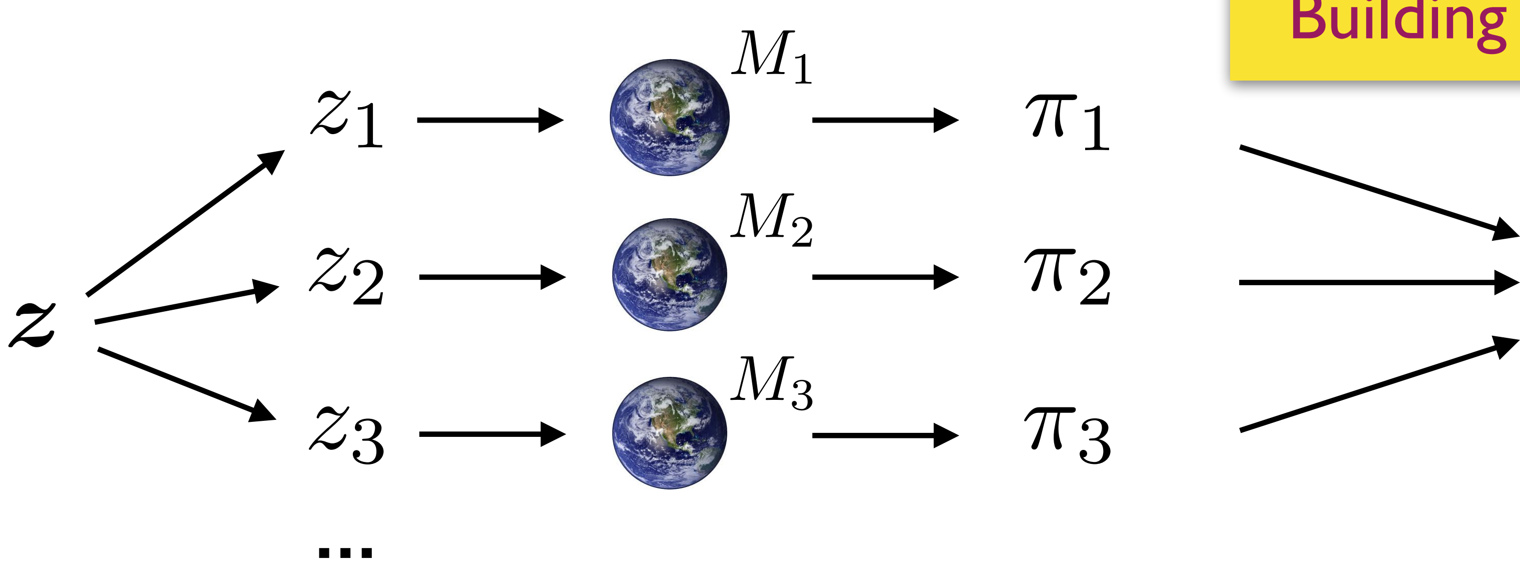
Building the internal model is essential.



policy learned with low cost
& robust to world change

Not Covered…

- Search, planning and game theory
- Bandit learning
- Imitation learning and Inverse RL
- RL Theory
- …

Further Reading

- David Silver's RL Course:
  https://www.davidsilver.uk/teaching/
- Deep RL course @ Berkeley:
  https://rail.eecs.berkeley.edu/deeprlcourse/
- Sutton and Barto book:
  http://incompleteideas.net/book/the-book-2nd.html
- OpenAI Gym platform:
  https://github.com/Farama-Foundation/Gymnasium

Thanks for your attention!
Discussions?

Acknowledgement: Many materials in this lecture are taken from
https://www.davidsilver.uk/teaching/
https://www.davidsilver.uk/wp-content/uploads/2020/03/AlphaGo-tutorial-slides_compressed.pdf